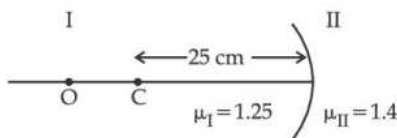


Single Choice Type

This section contains **20 Single choice questions**. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which **Only One** is correct.

1. Region I and II are separated by a spherical surface of radius 25 cm. an object is kept in region I at a distance of 40 cm from the surface. The distance of the image from the surface.

किसी वस्तु को क्षेत्र I में रखा गया है। क्षेत्र I तथा II को 25 cm त्रिज्या के गोलीय पृष्ठ द्वारा पृथक किया गया है। किसी बल्ब को क्षेत्र I में इस गोलीय पृष्ठ से 40 cm दूरी पर रखा गया है। प्रतिबिम्ब की पृष्ठ से दूरी होगी :



- (1) 9.52 cm (2) 18.23 cm (3) 55.44 cm (4) 37.58 cm

Ans. (4)

Sol. $\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$

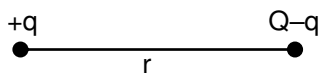
$\Rightarrow \frac{1.4}{v} - \frac{1.25}{-40} = \frac{1.4 - 1.25}{(-25)} \Rightarrow v = -37.58 \text{ cm}$

2. A certain charge Q is divided into two parts q and (Q - q). How should the charges Q and q be divided so that q and (Q - q). Placed at a certain distance apart experience maximum electrostatics repulsion.

किसी निश्चित आवेश Q को दो भागों q और (Q - q) में विभाजित किया गया है। Q और q को किस प्रकार विभाजित किया जाना चाहिए ताकि q और (Q - q) को एक-दूसरे से किसी निश्चित दूर पर रखे जाने पर ये परस्पर अधिकतम स्थिर विद्युत प्रतिकर्षण बल का अनुभव करें।

- (1) Q = 4q (2) Q = 3q (3) Q = $\frac{q}{2}$ (4) Q = 2q

Ans. (4)



Sol.

$F = \frac{Kq_1q_2}{r^2} = \frac{K(q)(Q - q)}{r^2}$

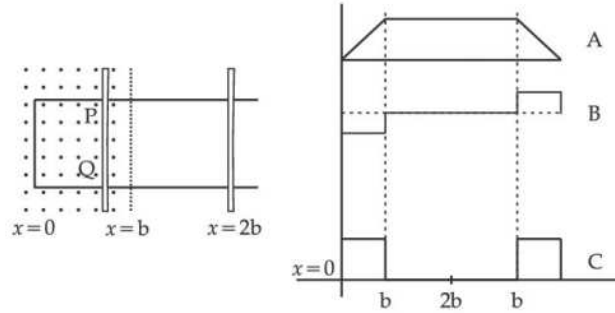
$\frac{dF}{dq} = 0$

$Q - 2q = 0$

$Q = 2q$

3. The arm PQ of a rectangular conductor is moving from $x = 0$ to $x = 2b$ outwards and then inwards from $x = 2b$ to $x = 0$ as shown in the figure. A uniform magnetic field perpendicular to the plane is acting from $x = 0$ to $x = b$. Identify the graph showing the variation of different quantities with distance.

किसी आयताकार चालक की भुजा PQ, $x = 0$ से $x = 2b$ तक बाहर की ओर फिर $x = 2b$ से $x = 0$ भीतर की ओर आरेख में दर्शाए अनुसार गतिमान है। $x = 0$ से $x = b$ तक तल के लम्बवत् कोई एकसमान चुम्बकीय क्षेत्र कार्यरत है। उस ग्राफ को पहचानिए जो दूरी के साथ विभिन्न राशियों के विचरण को दर्शाता है।



(1) A-Flux, B-Power dissipated, C-EMF

(2) A-EMF, B-Power dissipated, C-Flux

(3) A-Power dissipated, B-Flux, C-EMF

(4) A-Flux, B-EMF, C-Power dissipated

(1) A-फ्लक्स, B-शक्तिक्षय, C-EMF

(2) A-EMF, B-शक्तिक्षय, C-फ्लक्स

(3) A-शक्तिक्षय, B-फ्लक्स, C-EMF

(4) A-फ्लक्स, B-EMF, C-शक्तिक्षय

Ans. (4)

Sol.

$$\text{Flux} = \phi = B \cdot A$$

$$\Rightarrow B \times A \cos 0$$

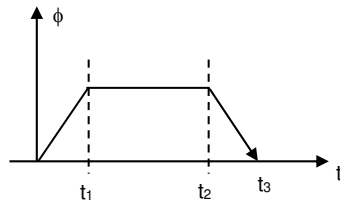
$$\text{Where } A = \ell vt$$

$$\phi = B\ell vt$$



One rod go at $x > b$ then ϕ stop changing this constant flux = $B\ell b$.

When rod come back and when $x < b$ flux start decreasing so graph $\phi v/st$

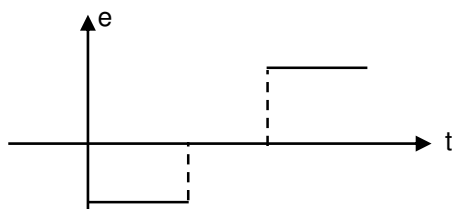


$b \rightarrow$ (ii)

$$e = \frac{-d\phi}{dt}; e = -\text{slope of } \phi - t \text{ graph}$$

In $0 - t_1$ graph slope +ve and constant so $e =$ negative and zero.

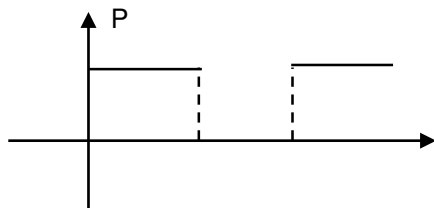
in $t_2 - t_3$ slope of $\phi - t$ is negative and constant so $e =$ positive and zero



$$\text{Power} = e^2/R$$

Resistance is only of rod so R of the circuit is constant

$$P = \frac{B^2 \ell^2 v^2}{R} = \text{constant}$$



4. If \vec{A} and \vec{B} are two vectors satisfying the relation $\vec{A} \cdot \vec{B} = |\vec{A} \times \vec{B}|$. Then the value of $|\vec{A} - \vec{B}|$ will be :

यदि \vec{A} और \vec{B} ऐसे दो सदिश हैं जो संबंध $\vec{A} \cdot \vec{B} = |\vec{A} \times \vec{B}|$ की पुष्टि करते हैं तब $|\vec{A} - \vec{B}|$ का मान होगा :

- (1) $\sqrt{A^2 + B^2 + 2AB}$ (2) $\sqrt{A^2 + B^2 - \sqrt{2}AB}$ (3) $\sqrt{A^2 + B^2}$ (4) $\sqrt{A^2 + B^2 + \sqrt{2}AB}$

Ans. (2)

Sol. $\vec{A} \cdot \vec{B} = |\vec{A} \times \vec{B}|$

$$\Rightarrow AB \cos \theta = AB \sin \theta$$

$$\therefore \theta = 45^\circ$$

$$\therefore |\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 - 2AB \cos 45^\circ}$$

$$= \sqrt{A^2 + B^2 - \sqrt{2}AB}$$

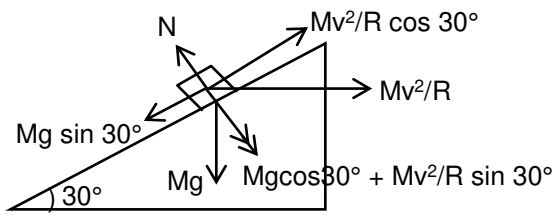
5. The normal reaction 'N' for a vehicle of 800 kg mass, negotiating a turn on a 30° banked road at maximum possible speed without skidding is _____ $\times 10^3$ kg m/s². (Given $\cos 30^\circ = 0.87$, $\mu_s = 0.2$)

अधिकतम संभावित चाल पर किसी झुकी हुई सड़क पर बिना फिसले 30° मोड़ को पार रकने के लिए किसी 800 kg द्रव्यमान के वाहन के लिए अभिलम्ब प्रतिक्रिया _____ $\times 10^3$ kg m/s² होगी।

- (1) 7.2 (2) 12.4 (3) 10.2 (4) 6.96

Ans. (3)

Sol.



Perpendicular to inclined plane

$$N = mg \cos 30^\circ + \frac{mv^2}{R} \sin 30^\circ$$

$$N - mg \cos 30^\circ = \frac{mv^2}{R} \sin 30^\circ \quad \dots(1)$$

along inclined plane

$$mg \sin 30^\circ + \mu_s N = \frac{mv^2}{R} \cos 30^\circ \quad \dots(2)$$

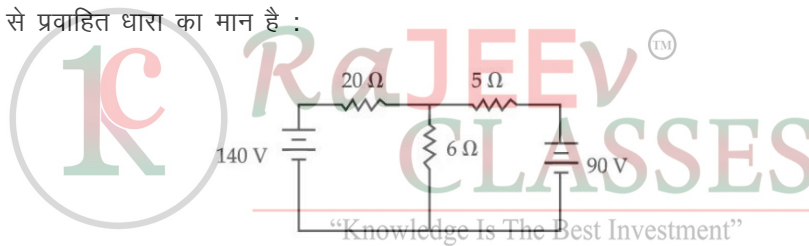
Dividing (1) by (2)

$$\frac{N - mg \cos 30^\circ}{mg \sin 30^\circ + \mu_s N} = \tan 30^\circ$$

$$N = 10.2 \times 10^3 \text{ kg m/s}^2$$

6. The value of current in the 6Ω resistance is :

6Ω के प्रतिरोध से प्रवाहित धारा का मान है :



(1) 4 A

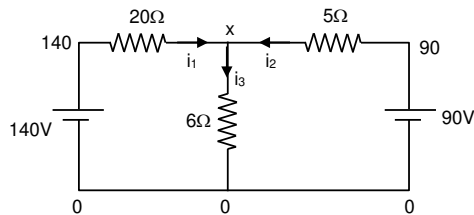
(2) 10 A

(3) 8 A

(4) 6 A

Ans. (2)

Sol.



Let potential at junction point = x

$$\text{By KCL } \sum i_{in} = 0 \Rightarrow \frac{140 - x}{20} + \frac{90 - x}{5} + \frac{0 - x}{6} = 0$$

$$x = 60 \text{ V}$$

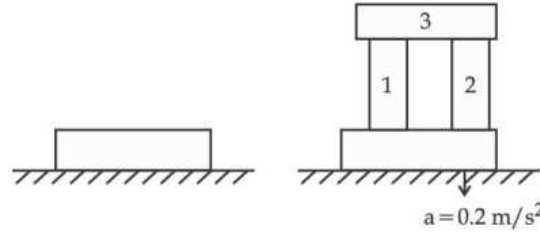
$$\text{So current } i_3 = \frac{x}{6} = \frac{60}{6} = 10 \text{ A}$$

7. A steel block of 10 kg rests on horizontal floor as shown. When three iron cylinders are placed on its as shown, the block and cylinders go down with an acceleration 0.2 m/s^2 . Then normal reaction R' by the floor if mass of the iron cylinders are equal and of 20 kg each, is _____ N.

(Take $g = 10 \text{ m/s}^2$ and $\mu_s = 0.2$)

आरेख में दर्शाए अनुसार किसी क्षैतिज फर्श पर 10 kg का कोई स्टील का गुटका रखा है। जब इस पर आयरन के तीन सिलिण्डर आरेख में दर्शाए अनुसार रखे जाते हैं, तो गुटका और सिलिण्डर 0.2 m/s^2 के त्वरण से नीचे जाते हैं। यदि आयरन के सिलिण्डरों में प्रत्येक का द्रव्यमान 20 kg हैं तब फर्श द्वारा अभिलम्ब प्रतिक्रिया R' का मान होगा।

($g = 10 \text{ m/s}^2$ and $\mu_s = 0.2$ लीजिए।)



(1) 686

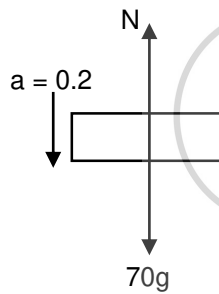
(2) 714

(3) 684

(4) 716

Ans. (1)

Sol.



$$70g - N = 70 \times 0.2$$

$$N = 700 - 14 ; N = 686$$

8. The amount of heat needed to raise the temperature of 4 moles of a rigid diatomic gas from 0°C to 50°C when no work is done is _____. (R is the universal gas constant)

किसी दृढ़ द्विपरमाणुक गैस के 4 मोल के ताप में 0°C से 50°C तक वृद्धि करने के लिए, आवश्यक ऊष्मा की मात्रा, जब कोई कार्य नहीं हो रहा है, होगी : (R सार्वत्रिक गैस नियतांक है।)

(1) 750 R

(2) 500 R

(3) 250 R

(4) 175 R

Ans. (2)

Sol. $n = 4$

$$\Delta T = 50\text{K}$$

$$C_v = \frac{5R}{2}$$

As $W = 0$. It means isochoric process

$$Q = \Delta U$$

$$= nC_v\Delta T = 4 \times \frac{5R}{2} \times 50 = 500 R$$

9. The radiation corresponding to $3 \rightarrow 2$ transition of a hydrogen atom falls on a gold surface to generate photoelectrons. These electrons are passed through a magnetic field of 5×10^{-4} T. Assume that the radius of the largest circular path followed by these electrons is 7 mm, the work function of the metal is : (Mass of electron = 9.1×10^{-31} kg)

फोटोइलेक्ट्रॉन उत्पन्न करने के लिए सोने (गोल्ड) के किसी पृष्ठ पर हाइड्रोजन परमाणु के $3 \rightarrow 2$ संक्रमण के तदनरूपी विकिरण को गिराया गया है। इन इलेक्ट्रॉनों को 5×10^{-4} T के चुम्बकीय क्षेत्र से गुजारा गया है। यह मानिये कि इन इलेक्ट्रॉनों द्वारा चले गए वृत्तीय पथ की अधिकतम त्रिज्या 7 mm है, तब धातु का कार्यफलन होगा : (इलेक्ट्रॉन का द्रव्यमान = 9.1×10^{-31} kg)

(1) 0.82 eV

(2) 0.16 eV

(3) 1.36 eV

(4) 1.88 eV

Ans. (1)

Sol.

$$E_p = 13.6 \left[\frac{1}{R_1^2} - \frac{1}{R_2^2} \right] \text{eV}$$

$$= 13.6 \left[\frac{1}{4} - \frac{1}{9} \right]$$

$$E_p = 1.89 \text{ eV}$$

For Gold plate

$$\phi = E_p - KE_{\text{max}}$$

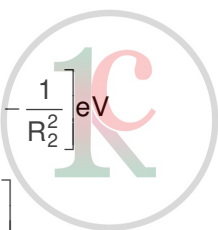
$$v = \frac{RqB}{m}$$

$$= \frac{7 \times 10^{-3} \times 1.6 \times 10^{-19} \times 5 \times 10^{-4}}{9.1 \times 10^{-31}} = 6.15 \times 10^5$$

$$K.E. = \frac{1}{2} mV^2$$

$$K.E. = \frac{1}{2} \times \frac{9.1 \times 10^{-31} \times (6.15 \times 10^5)^2}{1.6 \times 10^{-19}} \text{eV} = 1.075 \text{ eV}$$

$$\phi = 1.89 - 1.075 ; \phi = 0.82 \text{ eV}$$



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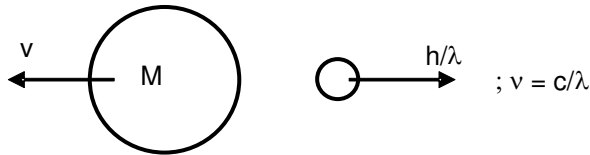
[Take 'c' as the speed of electromagnetic wave]

द्रव्यमान M का कोई नाभिक आवृत्ति 'v' के γ -किरण फोटॉन उत्सर्जित करता है। नाभिक की आन्तरिक ऊर्जा में होने वाली हानि है : [यहाँ 'c' विद्युत चुम्बकीय तरंग की चाल है]

- (1) 0 (2) $h\nu \left[1 + \frac{h\nu}{2Mc^2} \right]$ (3) $h\nu$ (4) $h\nu \left[1 - \frac{h\nu}{2Mc^2} \right]$

Ans. (2)

Sol.



$$Mv = \frac{h}{\lambda} = \left(\frac{h\nu}{c} \right)$$

$$\text{Loss of energy} = \frac{1}{2}Mv^2 + h\nu = \frac{1}{2} \frac{p^2}{M} + h\nu = \frac{1}{2M} \left(\frac{h\nu}{c} \right)^2 + h\nu = h\nu \left(1 + \frac{h\nu}{2Mc^2} \right)$$

11. Consider a mixture of gas molecule of types A, B and C having masses $m_A < m_B < m_C$. The ratio of their root mean square speeds at normal temperature and pressure is :

A, B और C प्रकार के अणुओं, जिनके द्रव्यमान $m_A < m_B < m_C$ के अनुसार हैं के मिश्रण पर विचार कीजिए। सामान्य ताप और दाब पर इनकी rms चालों का अनुपात होगा :

- “Knowledge Is The Best Investment”
- (1) $\frac{1}{v_A} > \frac{1}{v_B} > \frac{1}{v_C}$ (2) $v_A = v_B = v_C = 0$ (3) $v_A = v_B \neq v_C$ (4) $\frac{1}{v_A} < \frac{1}{v_B} < \frac{1}{v_C}$

Ans. (4)

Sol. $v_{rms} = \sqrt{\frac{3RT}{m}}$

12. AC voltage $V(t) = 20 \sin \omega t$ of frequency 50 Hz is applied to a parallel plate capacitor. The separation between the plates is 2 mm and the area is 1 m^2 . The amplitude of the oscillating displacement current for the applied AC voltage is _____. (Take $\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$)

किसी समान्तर पट्टिका संधारित्र पर 50 Hz आवृत्ति की कोई AC वोल्टता $V(t) = 20 \sin \omega t$ अनुप्रयुक्त की गयी है। पट्टिकाओं के बीच पृथकन 2 mm और क्षेत्रफल 1 m^2 है। आरोपित AC वोल्टता के लिए विस्थापन धारा का आयाम है _____। ($\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$)

- (1) 55.58 μA (2) 21.14 μA (3) 27.79 μA (4) 83.37 μA

Ans. (3)

Sol. $I_{dis} = \epsilon_0 \frac{d\phi_E}{dt}$

$$\phi_E = \frac{V(t)A}{d} = \frac{20 \sin 100\pi t \cdot 1}{2 \times 10^{-3}} = 10^4 \sin 100\pi t$$

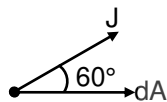
$$I_{dis} = \epsilon_0 \frac{d}{dt} (10^4 \sin 100\pi t) = 8.85 \times 10^{-12} \times 10^4 \times 100\pi \cos 100\pi t = 27.79 \mu A \cos 100\pi t$$

- 13.** A current of 5A is a passing through a non-linear magnesium wire of cross-section 0.04 m^2 . At every point the direction of current density is at an angle of 60° with the unit vector of area of cross-section. The magnitude of electric field at every point of the conductor is : (Resistivity of magnesium $\rho = 44 \times 10^{-8} \Omega\text{m}$)

0.04 m^2 अनुप्रस्थकाट क्षेत्रफल के मैग्नीशियम के किसी अरैखिक तार से 5A की धारा प्रवाहित हो रही है। तार के प्रत्येक बिन्दु पर धारा घनत्व की दिशा अनुप्रस्थकाट क्षेत्रफल के एकांक सदिश के साथ 60° का कोण बनाती है। इस चालक के प्रत्येक बिन्दु पर विद्युत क्षेत्र का परिमाण होगा : (मैग्नीशियम की प्रतिरोधकता $\rho = 44 \times 10^{-8} \Omega\text{m}$)

- (1) $11 \times 10^{-7} \text{ V/m}$ (2) $11 \times 10^{-5} \text{ V/m}$ (3) $11 \times 10^{-2} \text{ V/m}$ (4) $11 \times 10^{-3} \text{ V/m}$

Ans. (2)
Sol.



$$di = JdA \cos \theta = \frac{JdA}{2} ; i = \frac{EA}{\rho 2}$$

$$E = \frac{2pi}{A} = \frac{2 \times 44 \times 10^{-8} \times 5}{4 \times 10^{-2}} = 11 \times 10^{-5} \text{ V/m}$$

- 14.** The entropy of any system is given by $S = \alpha^2 \beta \ln \left[\frac{\mu k R}{J \beta^2} + 3 \right]$ where α and β are the constants. μ , J , k and R are no. of moles, mechanical equivalent of heat, Boltzmann constant and gas constant respectively. [Take $S = \frac{dQ}{T}$] Choose the incorrect options from the following :

- (1) S and α have different dimensions. (2) S , β , k and μR have the same dimensions.
(3) α and J have the same dimensions. (4) α and k have the same dimensions.

किसी निकाय की एन्ट्रॉपी इस प्रकार दी गयी है :

$$S = \alpha^2 \beta \ln \left[\frac{\mu k R}{J \beta^2} + 3 \right]$$

यहाँ α तथा β नियतांक है। μ , J , k और R क्रमशः मोलों की संख्या, ऊष्मा का यांत्रिक तुल्यांक, बोल्ड्मान स्थिरांक और गैस स्थिरांक हैं। [$S = \frac{dQ}{T}$ लीजिए] निम्नलिखित में से गलत विकल्प चुनिय :

- (1) S और α की असमान विमाएं है। (2) S , β , k और μR की समान विमाएं है।
(3) α और J की समान विमाएं है। (4) α और k की समान विमाएं है।

Ans. (4)

Sol. $S = \frac{Q}{\Delta T}$

$$[S] = \frac{ML^2T^{-2}}{K}$$

$$K = \frac{\text{Energy}}{T}$$

$$[K] = [S] = \frac{ML^2T^{-2}}{K}$$

$$[R] = \left[\frac{\text{Energy}}{nT} \right] = \frac{ML^2T^{-2}}{\text{molK}}$$

$$[J] = M^0L^0T^0$$

$$\text{Now, } [\mu KR] = [J\beta^2] \quad ; \quad (\text{mol}) \times \frac{ML^2T^{-2}}{K} \times \frac{ML^2T^{-2}}{\text{molK}} = [\beta^2]$$

$$[\beta] = ML^2T^{-2}K^{-1} \quad ; \quad [\alpha^2] = \left[\frac{S}{\beta} \right] = \frac{ML^2T^{-2}}{K \times ML^2T^{-2}K^{-1}} \quad ; \quad \alpha = M^0L^0T^0$$

15. A deuteron and an alpha particle having equal kinetic energy enter perpendicularly into a magnetic field. Let r_d and r_a be their respective radii of circular path. The value of $\frac{r_d}{r_a}$ is :

कोई ड्यूट्रॉन और कोई एल्फा कण, जिनकी गतिज ऊर्जा समान हैं किसी समान चुम्बकीय क्षेत्र में लम्बवत् प्रवेश करते हैं। मान लीजिए इनके वृत्तीय पथों की त्रिज्याएं क्रमशः r_d और r_a हैं, तब $\frac{r_d}{r_a}$ का मान होगा :

(1) $\frac{1}{\sqrt{2}}$

(2) $\sqrt{2}$

(3) 2

(4) 1

Ans. (2)

Sol. $r = \frac{mv}{qB} = \frac{\sqrt{2mK}}{qB}$

$$r \propto \frac{\sqrt{m}}{q}$$

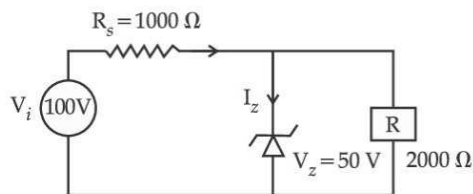
$$m_\alpha = 2m_d$$

$$q_\alpha = 2q_d$$

$$\frac{r_d}{r_\alpha} = \frac{\sqrt{m_d}}{q_d} \times \frac{2q_d}{\sqrt{2m_d}} = \sqrt{2}$$

16. For the circuit shown below, calculate the value of I_z :

नीचे दर्शाए गए परिपथ के लिए I_z का मान होगा :



(1) 0.15 A

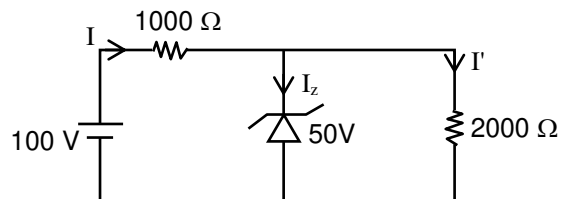
(2) 25 mA

(3) 0.05 A

(4) 0.1 A

Ans. (2)

Sol.



$$I = \frac{100 - 50}{1000} = 50 \text{ mA}$$

$$I' = \frac{50}{2000} = 0.025 \text{ mA}$$

$$I = I_z + I'$$

$$I_z = I - I' = 50 \text{ mA} - 25 \text{ mA} = 25 \text{ mA}$$

17. A radioactive material decays by simultaneous emissions of two particles with half lives of 1400 years and 700 years respectively. What will be the time after which one third of the material remains ? (Take $\ln 3 = 1.1$)

कोई रडियोएक्टिव पदार्थ क्रमशः 1400 वर्ष और 700 वर्ष की अर्धायु वाले दो कणों के समक्षणिक उत्सर्जनों द्वारा क्षयित हो रहा है। कितने समय के पश्चात् इस पदार्थ का एक तिहाई शेष बचेगा ? ($\ln 3 = 1.1$ लीजिए।)

(1) 700 years

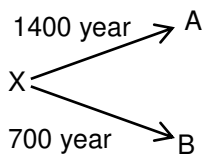
(2) 340 years

(3) 740 years

(4) 1110 years

Ans. (3)

Sol.



$$-\frac{dN_x}{dt} = \lambda_1 N_x + \lambda_2 N_x$$

Ans. (1)

Sol. \bar{g} (at any point) = $\bar{g}_{\text{Earth}} + \bar{g}_{\text{mars}}$. Since distance is large so $|\bar{g}| = |\bar{g}_E| = 10$.

As we move away from earth, It decrease to zero at a point where $\bar{g}_E + \bar{g}_M = 0$

Then it increase to $|\bar{g}| = |\bar{g}_M| = 4$ at mars surface.

20. The value of tension in a long thin metal wire has been changed from T_1 to T_2 . The lengths of the metal wire at two different values of tension T_1 and T_2 are l_1 and l_2 respectively. The actual length of the metal wire is :

किसी लम्बे पतल धातु के तार में तनाव के मान को T_1 से T_2 में परिवर्तित किया गया है। दो विभिन्न तनावों T_1 और T_2 पर धातु के तार की लम्बाईयों क्रमशः l_1 और l_2 हैं। तार की वास्तविक लम्बाई है :

(1) $\frac{T_1 l_1 - T_2 l_2}{T_1 - T_2}$

(2) $\frac{l_1 + l_2}{2}$

(3) $\frac{T_1 l_2 - T_2 l_1}{T_1 - T_2}$

(4) $\sqrt{T_1 T_2 l_1 l_2}$

Ans. (3)

Sol. Let initial length of rod be L_0 and Area A .

As $\frac{T}{A} = Y \frac{\Delta l}{l}$

So, $\frac{T_1}{A} = \frac{Y(L_1 - L_0)}{L_0}$

$\frac{T_2}{A} = \frac{Y(L_2 - L_0)}{L_0}$

Dividing

$\frac{T_1}{T_2} = \frac{L_1 - L_0}{L_2 - L_0}$

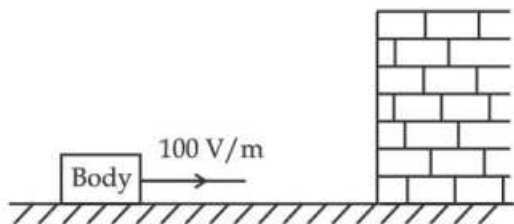
; $T_1 L_2 - T_1 L_0 = T_2 L_1 - T_2 L_0$; $L_0 = \frac{L_2 T_1 - L_1 T_2}{T_1 - T_2}$

Numerical Value Type

This section contains **10 Numerical value type questions.**

1. A body having specific charge $8 \mu\text{C/g}$ is resting on a frictionless plane at a distance 10 cm from the wall (as shown in the figure). It starts moving towards the wall when a uniform electric field of 100 V/m is applied horizontally towards the wall. If the collision of the body with the wall is perfectly elastic, the the time period of the motion will be _____ s.

आरेख में दर्शाए अनुसार $8 \mu\text{C/g}$ विशिष्ट आवेश का कोई पिण्ड किसी घर्षणहीन तल पर दीवार से 10 cm की दूरी पर रखा है। दीवार की ओर क्षैतिज दिशा में 100 V/m का एकसमान विद्युत क्षेत्र अनुप्रयुक्त किए जाने पर यह पिण्ड दीवार की ओर गति करने लगात है। यदि पिण्ड का दीवार से संघट्ट पूर्णतः प्रत्यास्थ है तब गति का आवर्तकाल _____ सेकण्ड होगा।



Ans. 1

Sol. $a = \frac{qE}{m} = \frac{8 \times 10^{-6}}{10^{-3}} \times 100 = 0.8 \text{ m/s}^2$

As electric field is switched on, ball first strikes to wall and returns back.
one oscillation.

Thus $s = ut_1 + \frac{1}{2}at_1^2$

$0.1 = \frac{1}{2} \times 0.8t_1^2$; $t_1 = \frac{1}{2} \text{ s}$

Thus time period $T = 2 \times \frac{1}{2} = 1 \text{ sec.}$

2. An object viewed from a near point distance of 25 cm, using a microscopic lens with magnification '6', gives an unresolved image. A resolved image is observed at infinite distance with a total magnification double the earlier using an eyepiece along with the given lens and a tube of length 0.6 m, if the focal length of the eyepiece is equal to _____ cm.

आवर्धन '6' के सूक्ष्मदर्शी लेंस का उपयोग करके निकट बिन्दु की दूरी 25 cm से किसी बिम्ब को देखने पर विभेदित प्रतिबिम्ब प्राप्त होता है। 0.6 m लम्बाई की नलिका और कोई अभिवृश्यक लेंस उपयोग करके अनन्त पर पहले से कुल दोगुने आवर्धन का विभेदित प्रतिबिम्ब देखा जाता है। इसके लिए अभिवृश्यक लेंस की फोकस दूरी _____ cm होगी।

Ans. 25

Sol. magnification of microscopic lens

$m = 1 + \frac{D}{f_0}$; $6 = 1 + \frac{25}{f_0}$; $f_0 = 5 \text{ cm}$

Magnification of compound microscope when image formed at infinity

$m = \frac{\ell}{f_0} \times \frac{D}{f_e}$

$12 = \frac{60}{5} \times \frac{25}{f_e}$; $f_e = 25 \text{ cm}$

3. A carrier wave $V_c(t) = 160 \sin(2\pi \times 10^6 t)$ volts is made to vary between $V_{\max} = 200 \text{ V}$ and $V_{\min} = 120 \text{ V}$ by a message signal $V_m(t) = A_m \sin(2\pi \times 10^3 t)$ volts. The peak voltage A_m of the modulating signal is _____

कोई वाहक तरंग $V_c(t) = 160 \sin(2\pi \times 10^6 t)$ वोल्ट किसी संदेश सिग्नल $V_m(t) = A_m \sin(2\pi \times 10^3 t)$ वोल्ट द्वारा माडुलित करने पर $V_{\max} = 200 \text{ V}$ और $V_{\min} = 120 \text{ V}$ के बीच विचरण करने लगती है। माडुलित सिग्नल की शिखर वोल्टता A_m _____ होगी।

Ans. 40

Sol. $V_{\max} = AC + A_m$

$\Rightarrow 200 = 160 + A_m$

$\Rightarrow A_m = 40$

4. The amplitude of wave disturbance propagating in the positive x-direction is given by $y = \frac{1}{(1+x)^2}$ at

time $t = 0$ and $y = \frac{1}{1+(x-2)^2}$ at $t = 1$ s, where x and y are in metres. The shape of wave doesnot

change during the propagation. The velocity of the wave will be _____ m/s.

धनात्मक x-दिशा में संचरित किसी तरंग विक्रोभ का आयाम समय $t = 0$ पर $y = \frac{1}{(1+x)^2}$ द्वारा तथा $t = 1$ s पर

$y = \frac{1}{1+(x-2)^2}$ द्वारा दर्शाया गया है, यहाँ x और y मीटर में हैं। संचरण के समय तरंग की आकृति में परिवर्तन नहीं

होता है। तरंग का वेग _____ m/s होगा।

Ans. 2

Sol. $x \rightarrow (x - vt)$

$$y = \frac{1}{1+(x-vt)^2}$$

$$\text{At } t = 0 ; y = \frac{1}{1+x^2} ; \text{ at } t = 1 \text{ sec} ; y = \frac{1}{1+(x-v)^2}$$

By comparing

$$V = 2 \text{ m/s}$$

5. A rod of mass M and length L is lying on a horizontal frictionless surface. A particle of mass ' m ' travelling along the surface hits at one end of the rod with a velocity ' u ' in a direction perpendicular to the rod. The collision is completely elastic. After collision, particle comes to rest. The ratio masses $\left(\frac{m}{M}\right)$

is $\frac{1}{x}$. The value of ' x ' will be _____.

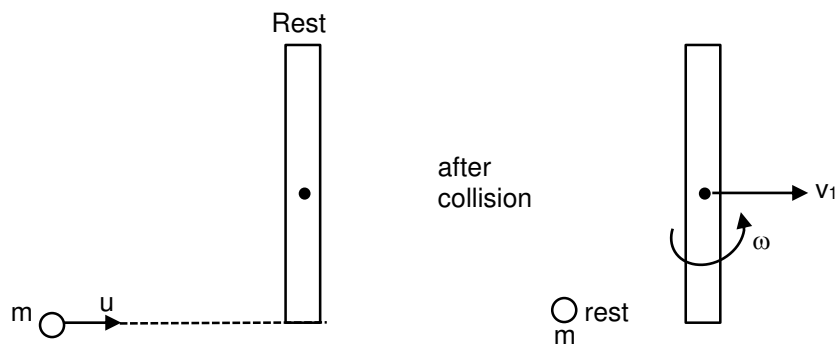
द्रव्यमान M और लम्बाई L की कोई छड़ किसी क्षैतिज घर्षणहीन पृष्ठ पर रखी है। पृष्ठ के अनुदिश गमन करता द्रव्यमान ' m ' का कोई कण इस छड़ के एक सिरे पर छड़ के लम्बवत् दिशा में वेग ' u ' से टक्कर मारता है। यह संघट्ट पूर्णत

प्रत्यास्थ है। संघट्ट के पश्चात् कण विराम में आ जाता है। द्रव्यमानों के अनुपात $\left(\frac{m}{M}\right)$ का मान $\frac{1}{x}$ है। यहाँ ' x ' का

मान _____ होगा।

Ans. 4

Sol.



Conservation of angular momentum about centre of mass of rod

$$mu \left(\frac{L}{2} \right) = \frac{ML^2}{12} (\omega) \quad \dots(i)$$

$$mu = Mv_1 \quad \dots(ii)$$

$$1 = \frac{v_1 + \omega \frac{L}{2}}{u} \quad \dots(iii)$$

Putting v_1 from (ii) and ωL from (i) in (iii)

$$u = \frac{m}{M}u + \frac{6mu}{2M}$$

$$1 = \frac{4m}{M} ; m/M = 1/4$$

6. The frequency of a car horn encountered a change from 400 Hz to 500 Hz, when the car approaches a vertical wall. If the speed of sound is 330 m/s. Then the speed of car is _____ km/h.

किसी कार के हॉर्न की आवृत्ति, जब वह किसी दीवार की ओर गमन करता है, तो 400 Hz से 500 Hz के परिवर्तन का सामना करती है। यदि ध्वनि की चाल 330 m/s है तो कार की चाल _____ km/h होगी।

Ans. 132

Sol. Frequency received by wall $f' = \left(\frac{v_s}{v_s - v} \right) f_0$

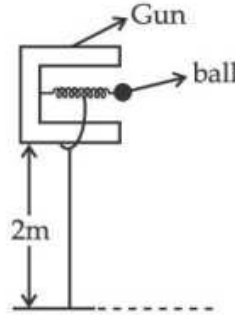
Reflected frequency received by man is $f'' = \left(\frac{v_s + v}{v_s} \right) f'$

$$\Rightarrow f'' = \left(\frac{v_s + v}{v_s} \right) \left(\frac{v_s}{v_s - v} \right) f_0 \Rightarrow f'' = \left(\frac{v_s + v}{v_s - v} \right) f_0 \Rightarrow 500 = \left(\frac{330 + v}{330 - v} \right) 400$$

$$\Rightarrow v = \frac{330}{9} \times \frac{18}{5} = 132 \text{ km/hr}$$

7. In a spring gun having spring constant 100 N/m a small ball 'B' of mass 100 g is put in its barrel (as shown in figure) by compressing the spring through 0.05 m . There should be a box placed at a distance 'd' on the ground so that the ball falls in it. If the ball leaves the gun horizontally at a height of 2 m above the ground. The value of d is _____ m. ($g = 10 \text{ m/s}^2$)

कमानी स्थिरांक 100 N/m की किसी कमानीदार बन्दूक में द्रव्यमान 100 g की कोई छोटी गेंद 'B' इसकी बैरल में (आरेख देखिए) कमानी को 0.05 m संपीडित करके रखी है। धरती पर दूरी 'd' पर किसी बॉक्स को रखा जाता है, ताकि यह गेंद उस बॉक्स में गिरे। यदि गेंद धरती से 2 m ऊँचाई पर बन्दूक से क्षैतिजतः निकलती है तो d का मान _____ m होगा। ($g = 10 \text{ m/s}^2$)



Ans. 1

Sol. By energy conservation

$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2 \Rightarrow v = x\sqrt{\frac{k}{m}} \quad v = 0.05 \times \sqrt{\frac{100}{0.1}} = 0.5\sqrt{10} \text{ m/s}$$

$$\text{Time of flight of ball } T = \sqrt{\frac{2H}{g}} = \sqrt{\frac{2 \times 2}{10}} = \frac{2}{\sqrt{10}} \text{ sec}$$

Range of ball $s = ut$

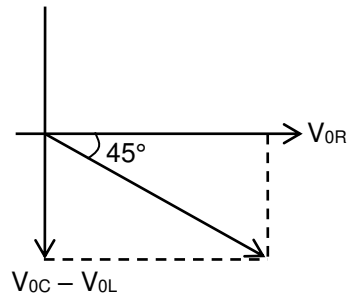
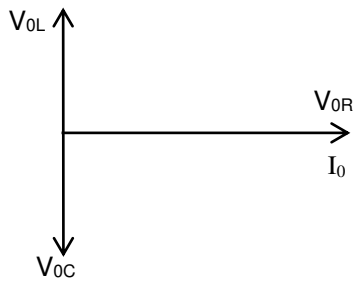
$$d = 0.5\sqrt{10} \times \left(\frac{2}{\sqrt{10}}\right) = 1 \text{ m}$$

8. In an LCR series circuit, an inductor 30 mH and a resistor 1Ω are connected to an AC source of angular frequency 300 rad/s . The value of capacitance for which, the current leads the voltage by 45° is $\frac{1}{x} \times 10^{-3} \text{ F}$. Then the value of x is _____.

किसी LCR श्रेणी परिपथ में 30 mH का कोई प्रेरक तथा 1Ω को कोई प्रतिरोधक 300 rad/s कोणीय आवृत्ति के किसी AC स्रोत से संयोजित हैं। धारिता का वह मान जिसके लिए धारा, वोल्टता से 45° अग्र हो $\frac{1}{x} \times 10^{-3} \text{ F}$ हैं। तब x का मान _____ होगा।

Ans. 3

Sol.



$$\tan 45^\circ = \frac{V_{0C} - V_{0L}}{V_{0R}} = \frac{X_C - X_L}{R}$$

$$R = \frac{1}{\omega C} - \omega L$$

$$1\Omega = \frac{1}{300C} - 30 \times 10^{-3} \times 300$$

$$\Rightarrow C = \frac{1}{3} \times 10^{-3} \text{ F}$$

9. A circular disc reaches from top to bottom of an inclined plane of length 'L'. When it slips down the plane, it takes time 't₁'. When it rolls down the plane, it takes time t₂. The value of $\frac{t_2}{t_1}$ is $\sqrt{\frac{3}{x}}$. Then value of x will be _____.

कोई वृत्तीय डिस्क, लम्बाई 'L' के किसी आनत तल के शीर्ष से उसकी तली पर पहुँचती है। जब यह तल पर नीचे सरकती है तो 't₁' समय लेती है। जब यह तल पर नीचे की ओर लुढ़कती है तो 't₂' समय लेती है। $\frac{t_2}{t_1}$ का मान $\sqrt{\frac{3}{x}}$ है। x का मान _____ होगा।

Ans. 2

Sol. When disc slides $a_1 = g \sin \theta$ So $S = ut_1 + \frac{1}{2} a_1 t_1^2 = \frac{1}{2} g \sin \theta \cdot t_1^2 \dots(1)$

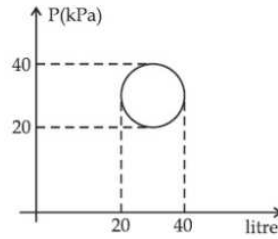
$$\text{When disc do pure rolling } a_2 = \frac{g \sin \theta}{1 + k^2/R^2} = \frac{g \sin \theta}{1 + 1/2} = \frac{2}{3} g \sin \theta$$

$$\text{So } S = ut_2 + \frac{1}{2} a_2 t_2^2 = \frac{1}{2} \cdot \frac{2}{3} g \sin \theta \cdot t_2^2 \dots(2)$$

From (1) & (2)

$$\frac{t_2}{t_1} = \sqrt{\frac{3}{2}}$$

10. In the reported figure, heat energy absorbed by a system in going through a cyclic process is _____ π J.
दिए गए आरेख में चक्रीय प्रक्रम से गुजरने में किसी निकाय द्वारा अवशोषित ऊष्मीय ऊर्जा का मान _____ π J होता है।



Ans. 100

Sol. $\Delta Q = W + \Delta U = W = \text{area enclosed by the curve}$

$$\Delta Q = \pi ab$$

$$= \pi \left[\frac{40-20}{2} \times 10^3 \right] \times \left[\frac{40-20}{2} \times 10^{-3} \right]$$

$$= 100 \pi \text{ Joule}$$



CHEMISTRY

Section : Chemistry Section A

1. Compound A is converted to B on reaction with CHCl_3 and KOH . The compound B is toxic and can be decomposed by C. A, B and C respectively are :

- (1) Primary amine, isonitrile compound, conc. HCl
- (2) Secondary amine, nitrile compound, conc. NaOH
- (3) Primary amine, nitrile compound, conc. HCl
- (4) Secondary amine, isonitrile compound, conc. NaOH

Ans. (1)

Sol. Only 1° amines give carbylamines reaction



2. Given below are two statements. One is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A : Sharp glass becomes smooth on heating it upto its melting point.

Reason R : The viscosity of glass decreases on melting.

Choose the most appropriate answer from the options given below :

- (1) A is false but R is true.
- (2) Both A and R are true but R is NOT the correct explanation of A.
- (3) A is true but R is false.
- (4) Both A and R are true and R is the correct explanation of A.

Ans. (2)

Sol. On heating viscosity decreases, but does not have any relation with smoothing of glass on heating.

3. Given below are two statements : One is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A : The dihedral angles in H_2O_2 in gaseous phase is 90.2° and in solid phase is 111.5° .

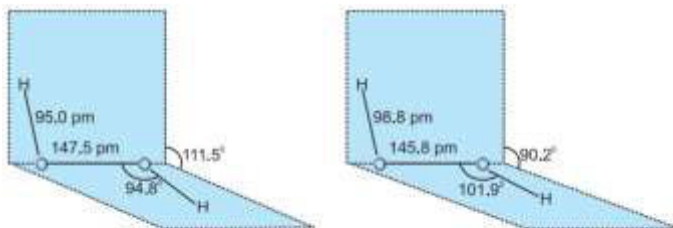
Reason R : The change in dihedral angle in solid and gaseous phase is due to the difference in the intermolecular forces.

Choose the most appropriate answer from the options given below for A and R.

- (1) A is correct but R is not correct.
- (2) Both A and R are correct and R is the correct explanation of A.
- (3) A is not correct but R is correct.
- (4) Both A and R are correct but R is not the correct explanation of A.

Ans. (3)

Sol.



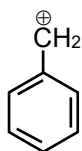
(a) Gas phase

(b) Solid phase

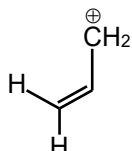
(a) H_2O_2 structure in gas phase, dihedral angle is 111.5° . (b) H_2O_2 structure in solid phase at 110 K, dihedral angle is 90.2° .

The dihedral angle of H_2O_2 in gaseous phase is approximate 111.5° . While dihedral angle in solid H_2O_2 is affected by hydrogen bonding and it is 90.2° in solid state.

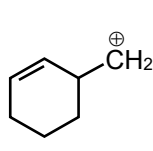
4.



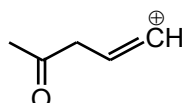
(A)



(B)



(C)



(D)

Among the given species the Resonance stabilised carbocations are :

(1) (A), (B) and (C) only

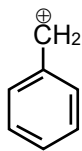
(2) (A) and (B) only

(3) (C) and (D) only

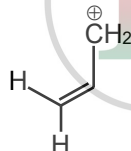
(4) (A), (B) and (D) only

Ans.

(2)

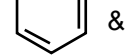


(A)

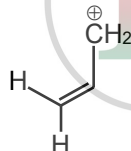


(B)

Sol.



&



are resonance stabilised carbocations.

5. Identify the incorrect statement from the following :

(1) Starch is a polymer of α -D glucose

(2) Glycogen is called as animal starch

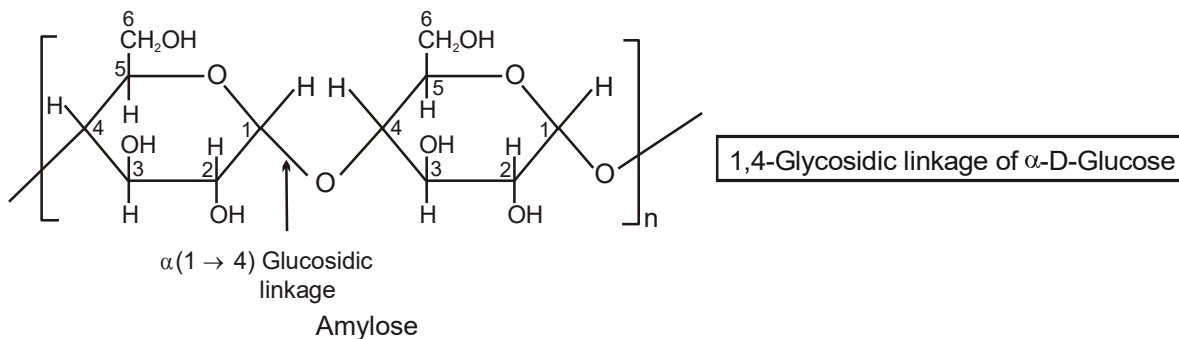
(3) β -Glycosidic linkage makes cellulose polymer

(4) Amylose is a branched chain polymer of glucose

Ans.

(4)

Sol. The **amylose** molecule is made up of D-glucose unit joined by α -glycosidic linkages between C-1 of one glucose unit and C-4 of the next glucose unit.



6. Green Chemistry in day-to-day life is in the use of :

- (1) Liquefied CO_2 for dry cleaning of clothes (2) Large amount of water alone for washing clothes
(3) Tetrachloroethene for laundry (4) Chlorine for bleaching of paper

Ans. (1)

Sol. $\text{CCl}_2=\text{CCl}_2$ was earlier used as solvent for dry cleaning agent but it is carcinogen. So liquid CO_2 is used. Replacement of halogenated solvent by liquid CO_2 will result in less harm to ground water.

7. The set in which compounds have different nature is :

- (1) $\text{B}(\text{OH})_3$ and $\text{Al}(\text{OH})_3$ (2) $\text{B}(\text{OH})_3$ and H_3PO_3
(3) $\text{Be}(\text{OH})_2$ and $\text{Al}(\text{OH})_3$ (4) NaOH and $\text{Ca}(\text{OH})_2$

Ans. (1)

Sol. $\text{B}(\text{OH})_3$ is H_3BO_3 is acidic in nature.

$\text{Al}(\text{OH})_3$ is amphoteric in nature.

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8. The species given below that does NOT show disproportionation reaction is :

- (1) BrO^- (2) BrO_4^- (3) BrO_3^- (4) BrO_2^-

Ans. (2)

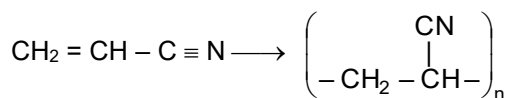
Sol. In BrO_4^- , Br is in maximum oxidation state. So it can only reduce.

9. Orlon fibres are made up of :

- (1) Polyacrylonitrile (2) Cellulose (3) Polyamide (4) Polyesters

Ans. (1)

Sol. Orlon is a polymer of acrylonitrile also known as PAN



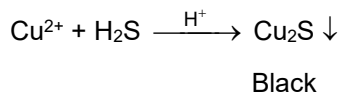
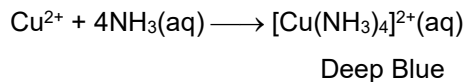
Acrylonitrile

PAN or orlon

10. An inorganic compound 'X' on treatment with concentrated H_2SO_4 produces brown fumes and gives dark brown ring with FeSO_4 in presence of concentrated H_2SO_4 . Also compound 'X' gives precipitated 'Y', when its solution dilute HCl is treated with H_2S gas. The precipitate 'Y' on treatment with concentrated HNO_3 followed by excess of NH_4OH further gives deep blue coloured solution, Compound 'X' is :
- (1) $\text{Co}(\text{NO}_3)_2$ (2) $\text{Cu}(\text{NO}_3)_2$ (3) $\text{Pb}(\text{NO}_3)_2$ (4) $\text{Pb}(\text{NO}_2)_2$

Ans. (2)

Sol. Nitrates give brown ring test.



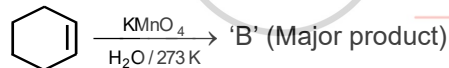
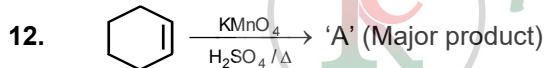
11. The correct order of intensity of colors of the compounds is :

- (1) $[\text{Ni}(\text{H}_2\text{O})_6]^{2+} > [\text{NiCl}_4]^{2-} > [\text{NiCN}]_4]^{2-}$ (2) $[\text{NiCl}_4]^{2-} > [\text{Ni}(\text{H}_2\text{O})_6]^{2+} > [\text{Ni}(\text{CN})_4]^{2-}$
 (3) $[\text{NiCl}_4]^{2-} > [\text{NiCN}]_4]^{2-} > [\text{Ni}(\text{H}_2\text{O})_6]^{2+}$ (4) $[\text{Ni}(\text{CN})_4]^{2-} > [\text{NiCl}_4]^{2-} > [\text{Ni}(\text{H}_2\text{O})_6]^{2+}$

Ans. (2)

Sol. As all complexes are Ni^{2+} so stronger the ligand greater is splitting and lighter is colour. Order of strength of ligand $\text{Cl}^- < \text{H}_2\text{O} < \text{CN}^-$.

So, order of intensity of colour $[\text{NiCl}_4]^{2-} > [\text{Ni}(\text{H}_2\text{O})_6]^{2+} > [\text{Ni}(\text{CN})_4]^{2-}$.

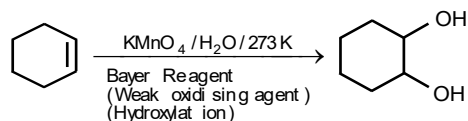
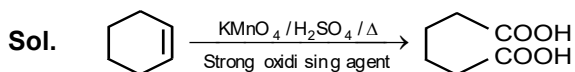


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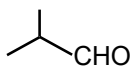
For above chemical reactions, identify the correct statement from the following.

- (1) Compound 'A' is diol and compound 'B' is dicarboxylic acid.
 (2) Both compound 'A' and compound 'B' are dicarboxylic acids.
 (3) Both compound 'A' and compound 'B' are diols.
 (4) Compound 'A' is dicarboxylic acid and compound 'B' is diol.

Ans. (4)



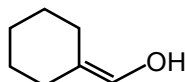
13.



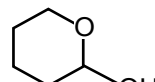
(I)



(II)



(III)



(IV)

Which among the above compound/s does/do not form silver mirror when treated with Tollen's reagent?

(1) (I), (III) and (IV) only

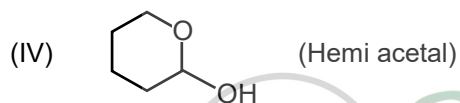
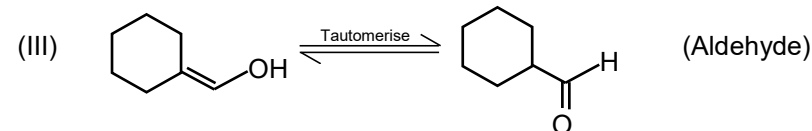
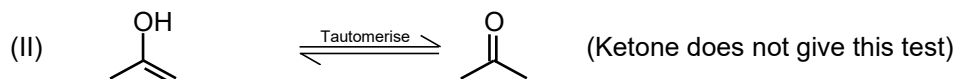
(2) Only (II)

(3) (III) and (IV) only

(4) Only (IV)

Ans. (2)**Sol.** Aldehyde & hemiacetal group form silver mirror when treated with Tollen's reagent.

Hence compound (I), (III) and (IV) gives this test.



14. The metal that can be purified economically by fractional distillation method is :

(1) Cu

(2) Zn

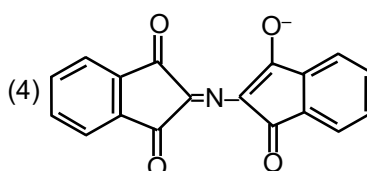
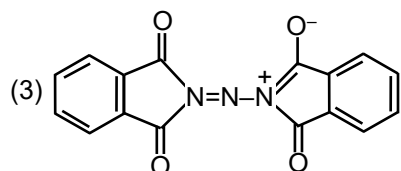
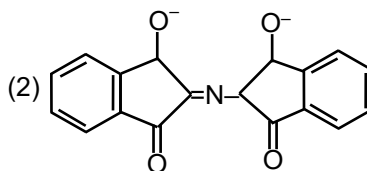
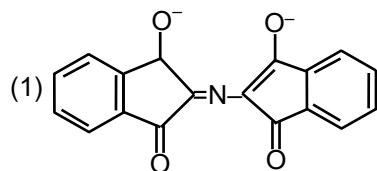
(3) Fe

(4) Ni

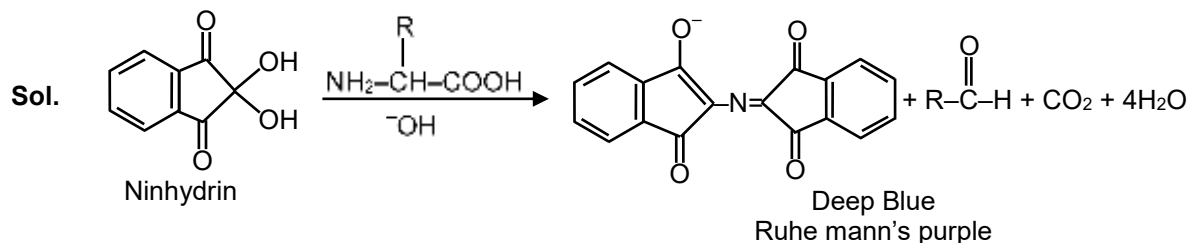
Ans. (2)**Sol.** Fractional distillation process utilizes the boiling point difference between metal and that of impurity.

Using this process, crude zinc containing Cd, Fe and Pb as impurities can be refined.

15. The correct structure of Rhumann's Purple, the compound formed in the reaction of ninhydrin with proteins is :



Ans. (4)

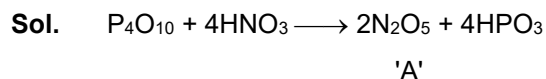


Ninhydrin is useful for identification of α -amino acid which react with ninhydrin and give deep blue colour.

16. Chemical nature of the nitrogen oxide compound obtained from a reaction of concentrated nitric and P_4O_{10} (in 4 : 1 ratio) is :

- (1) amphoteric (2) basic (3) neutral (4) acidic

Ans. (4)



Nature of oxide 'A' is "acidic".

17. According to the valence bond theory the hybridization of central metal atom is dsp^2 for which one of the following compound ?

- (1) $Na_2[NiCl_4]$ (2) $[Ni(CO)_4]$ (3) $K_2[Ni(CN)_4]$ (4) $NiCl_2 \cdot 6H_2O$

Ans. (3)

Sol.

	Complex	Hybridisation
(1)	$Na_2[NiCl_4]$	sp^3
(2)	$[Ni(CO)_4]$	sp^3
(3)	$K_2[Ni(CN)_4]$	dsp^2
(4)	$NiCl_2 \cdot 6H_2O$	sp^3d^2

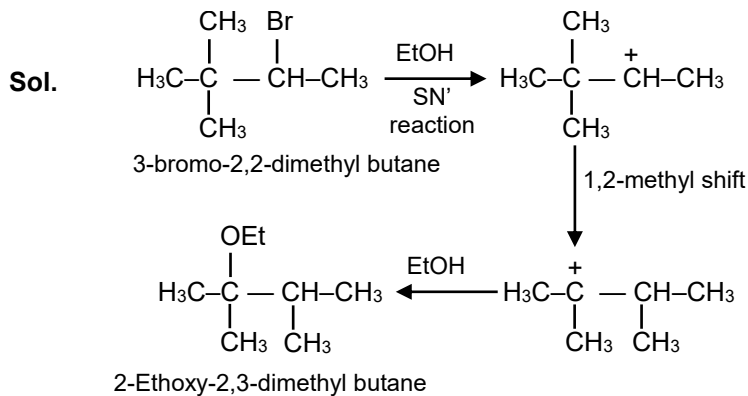
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18. In the given reaction 3-Bromo-2,2-dimethyl butane $\xrightarrow{C_2H_5OH}$ (Major Product) 'A'

Product A is :

- (1) 2-Ethoxy-3,3-dimethyl butane (2) 2-Hydroxy-3,3-dimethyl butane
(3) 2-Ethoxy-2,3-dimethyl butane (4) 1-Ethoxy-3,3-dimethyl butane

Ans. (3)



19. The conditions give below are in the context of observing Tyndall effect in colloidal solutions :

- The diameter of the colloidal particles is comparable to the wavelength of light used.
- The diameter of the colloidal is much smaller than the wavelength of light used.
- The diameter of the colloidal particles is much larger than the wavelength of light used.
- The refractive indices of the dispersed phase and the dispersion medium are comparable.
- The dispersed phase has a very different refractive index from the dispersion medium.

Choose the most appropriate conditions from the options given below :

- (1) (C) and (D) only (2) (A) and (D) only (3) (B) and (E) only (4) (A) and (E) only

Ans. (4)

Sol. The conditions give below are in the context of observing Tyndall effect in colloidal solutions :

- The diameter of the colloidal particles is comparable to the wavelength of light used.
- The refractive index of the dispersed phase and dispersion medium differ greatly in magnitude.

20. A s-block element (M) reacts with oxygen to form an oxide of the formula MO_2 . The oxide is pale yellow in colour and paramagnetic. The element (M) is :

- (1) Mg (2) K (3) Na (4) Ca

Ans. (2)

Sol. $\text{K} + \text{O}_2 (\text{excess}) \longrightarrow \text{KO}_2$

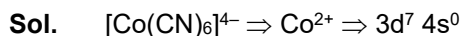
Potassium on reaction with excess oxygen give superoxide

Section : Chemistry Section B

1. The spin-only magnetic moment value for the complex $[\text{Co}(\text{CN})_6]^{4-}$ is.....BM.

[At. no. of Co = 27]

Ans. (2)



So number of unpaired electrons = 1.

$$\mu = \sqrt{n(n+2)} = \sqrt{3}$$

$$\mu = 1.73 \text{ BM} \approx 2 \text{ BM.}$$

Note : Answer given by NTA (2) but Reso answer should be (1.73). Since nearest integer not mentioned in the question.

2. An average person needs about 10000 kJ energy per day. The amount of glucose (molar mass = 180.0 g mol⁻¹) needed to meet this energy requirement is.g.

(Nearest integer)

(Use : $\Delta_c H(\text{glucose}) = -2700 \text{ kJ mol}^{-1}$)

Ans. (667)



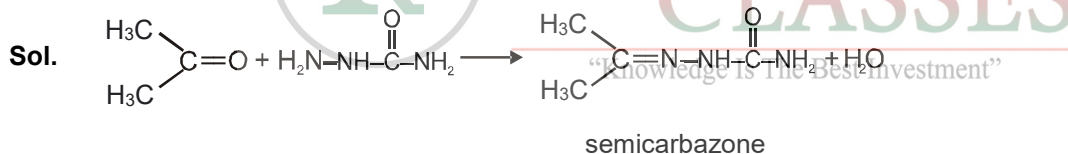
Glucose

No. of mole of glucose require for production of 10,000 KJ heat is = $\frac{10,000}{2700}$ mole.

Total mass of glucose = $\frac{10,000}{2700} \times 180 = 666.67 \text{ gram.}$

3. The number of nitrogen atoms in a semicarbazone molecule of acetone is.....

Ans. (3)



4. The inactivation rate of a viral preparation is proportional to the amount of virus. In the first minute after preparation, 10% of the virus is inactivated. The rate constant for viral inactivation is..... $\times 10^{-3} \text{ min}^{-1}$. (Nearest integer)

[Use : $\ln 10 = 2.303$; $\log_{10} 3 = 0.477$; Property of logarithm : $\log x^y = y \log x$]

Ans. (106)

Sol. $K = \frac{1}{t} \ln \left[\frac{a}{a-x} \right]$

$$= \frac{2.303}{1} \log \left[\frac{100}{90} \right]$$

$$= \frac{2.303}{1} \log [\log 10 - 2 \log 3]$$

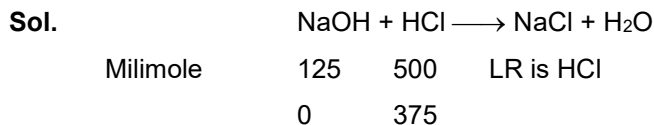
$$= 2.303 [1 - 2 \times 0.477]$$

$$= 2.303 \times 0.046 = 0.1059$$

$$= 105.9 \times 10^{-3} = 106 \times 10^{-3}$$

5. 250 mL of 0.5 M NaOH was added to 500 mL of 1 M HCl. The number of unreacted HCl molecules in the solution after complete reaction is..... $\times 10^{21}$. (Nearest integer)
($N_A = 6.022 \times 10^{23}$)

Ans. (226)



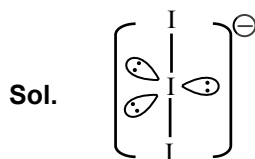
No. of molecules of HCl remaining unreacted

$$= 375 \times 10^{-3} \times 6.022 \times 10^{23}$$

$$= 2.258 \times 10^{23} = 225.8 \times 10^{21} \approx 226 \times 10^{21}$$

6. The number of lone pairs of electrons on the central I atom in I_3^- is.....

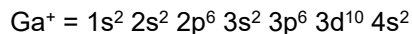
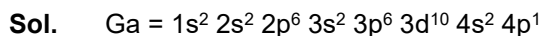
Ans. (3)



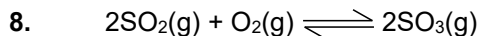
Total lone pair on central atom = 3.

7. The Azimuthal quantum number for the valence electrons Ga^+ ion is.....
(Atomic number of Ga = 31)

Ans. (0)



Azimuthal Quantum number (ℓ) for valence shell electron is 0.



In an equilibrium mixture, the partial pressures are

$P_{\text{SO}_3} = 43 \text{ kPa}$; $P_{\text{O}_2} = 530 \text{ Pa}$ and $P_{\text{SO}_2} = 45 \text{ kPa}$.

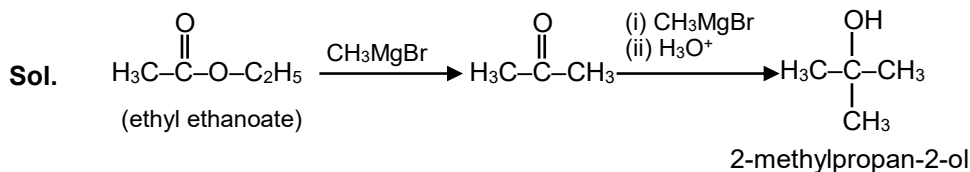
The equilibrium constant $K_P = \dots\dots\dots \times 10^{-2}$. (Nearest integer)

Ans. (172)

Sol.
$$K_P = \frac{(P_{\text{SO}_3})^2}{(P_{\text{SO}_2})^2(P_{\text{O}_2})} = \frac{(43)^2}{(45)^2(0.53)} = 1.72 \approx 172 \times 10^{-2}$$

9. To synthesis 1.0 mole of 2-methylpropan-2-ol from Ethylethanoate.....equivalents of CH_3MgBr reagent will be required. (Integer value)

Ans. (2)



10. At 20°C , the vapour of benzene is 70 torr and that of methyl benzene is 20 torr. The mole fraction of benzene in the vapour phase at 20°C above an equimolar mixture of benzene and methyl benzene is..... $\times 10^{-2}$. (Nearest integer)

Ans. (78)

Sol.
$$P_{\text{Total}} = P_{\text{Benzene}}^{\circ} X_{\text{Benzene}} + P_{\text{Toluene}}^{\circ} X_{\text{Toluene}}$$

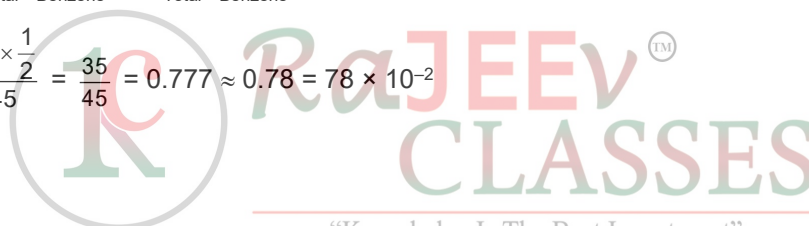
$$= (70) \frac{1}{2} + (20) \frac{1}{2}$$

$$= 35 + 10$$

$$= 45$$

$$P_{\text{Benzene}} = P_{\text{Total}} Y_{\text{Benzene}} = P_{\text{Total}}^{\circ} Y_{\text{Benzene}}$$

$$Y_{\text{Benzene}} = \frac{70 \times \frac{1}{2}}{45} = \frac{35}{45} = 0.777 \approx 0.78 = 78 \times 10^{-2}$$



MATHEMATICS

1. The value of the integral $\int_{-1}^1 \log_e(\sqrt{1-x} + \sqrt{1+x}) dx$ is equal to :

समाकलन $\int_{-1}^1 \log_e(\sqrt{1-x} + \sqrt{1+x}) dx$ का मान बराबर है :

- (1) $\log_e 2 + \frac{\pi}{2} - 1$ (2) $2\log_e 2 + \frac{\pi}{4} - 1$ (3) $2\log_e 2 + \frac{\pi}{2} - \frac{1}{2}$ (4) $\frac{1}{2}\log_e 2 + \frac{\pi}{4} - \frac{3}{2}$

Ans. (1)

Sol. $f(x) = \ln(\sqrt{1-x} + \sqrt{1+x})$ $x \in [-1, 1]$ is an even function

$$\Rightarrow I = 2 \int_0^1 \ln(\sqrt{1-x} + \sqrt{1+x}) dx$$

Put $x = \cos 2\theta \Rightarrow dx = -2\sin 2\theta d\theta$

$\therefore \cos 2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

$$\Rightarrow I = -4 \int_0^{\frac{\pi}{4}} \left[\ln\left\{(\sin\theta + \cos\theta)\sqrt{2}\right\} \right] \sin 2\theta d\theta$$

$$= 4 \int_0^{\frac{\pi}{4}} \left[\ln\left\{(\sin\theta + \cos\theta)\sqrt{2}\right\} \right] \sin 2\theta d\theta$$

$$= 4 \int_0^{\frac{\pi}{4}} \ln(\sin\theta + \cos\theta) \sin 2\theta d\theta + 4 \ln\sqrt{2} \int_0^{\frac{\pi}{4}} \sin 2\theta d\theta$$

$$= 4 \left[-\ln(\sin\theta + \cos\theta) \frac{\cos 2\theta}{2} \Big|_0^{\frac{\pi}{4}} + \int_0^{\frac{\pi}{4}} \frac{\cos\theta - \sin\theta}{\sin\theta + \cos\theta} \cdot \frac{\cos 2\theta}{2} d\theta \right] + 4 \ln\sqrt{2} \left(-\frac{\cos 2\theta}{2} \right) \Big|_0^{\frac{\pi}{4}}$$

$$= 4 \left[0 + \frac{1}{2} \int_0^{\frac{\pi}{4}} (\cos\theta - \sin\theta)^2 d\theta \right] + 4 \ln\sqrt{2} \left(0 + \frac{1}{2} \right)$$

$$= 4 \left[0 + \frac{1}{2} \int_0^{\frac{\pi}{4}} (1 - \sin 2\theta) d\theta \right] + 2 \ln\sqrt{2} = 2 \left[\theta + \frac{\cos 2\theta}{2} \right]_0^{\frac{\pi}{4}} + \ln 2$$

$$= 2 \left[\frac{\pi}{4} - \frac{1}{2} \right] + \ln 2 = \frac{\pi}{2} - 1 + \ln 2$$

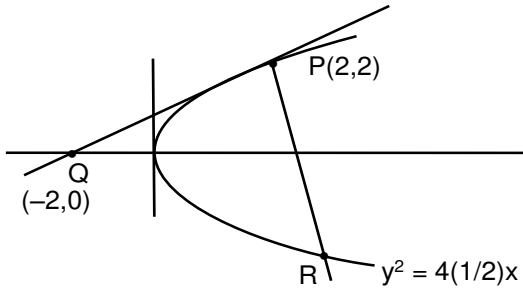
2. Let the tangent to the parabola $S : y^2 = 2x$ at the point $P(2, 2)$ meet the x -axis at Q and normal at it meet the parabola S at the point R . Then the area (in sq. units) of the triangle PQR is equal to :

माना परवलय $S : y^2 = 2x$ के बिन्दु $P(2, 2)$ पर स्पर्श रेखा x -अक्ष को बिन्दु Q पर मिलती है तथा इस बिन्दु पर अभिलम्ब परवलय S को बिन्दु R पर मिलता है। तो त्रिभुज PQR का क्षेत्रफल (वर्ग इकाईयों में) बराबर है—

- (1) $\frac{35}{2}$ (2) $\frac{25}{2}$ (3) $\frac{15}{2}$ (4) 25

Ans. (2)

Sol.



Equation of tangent at $P(2,2)$ is $T = 0$

$$2y = x + 2$$

So, $Q = (-2, 0)$

$$2at_1 = 2 \Rightarrow t_1 = 2$$

$$t_2 = -t_1 - \frac{2}{t_1} = -2 - \frac{2}{2} = -3$$

$$\therefore R\left(\frac{1}{2}(-3)^2, 2\left(\frac{1}{2}\right)(-3)\right) = \left(\frac{9}{2}, -3\right)$$

$$\text{Area of } \triangle PQR = \frac{1}{2} \begin{vmatrix} 2 & 2 & 1 \\ -2 & 0 & 1 \\ \frac{9}{2} & -3 & 1 \end{vmatrix}$$

$$= \frac{1}{2} [2(0+3) - 2(-2 - 9/2) + 1(6 - 0)] = \frac{1}{2} [6 + 4 + 9 + 6] = \frac{25}{2} \text{ sq. unit.}$$



3. The coefficient of x^{256} in the expansion of $(1-x)^{101}(x^2+x+1)^{100}$ is :

$(1-x)^{101}(x^2+x+1)^{100}$ के प्रसार में x^{256} का गुणांक है :

- (1) ${}^{100}C_{15}$ (2) ${}^{100}C_{16}$ (3) ${}^{100}C_{16}$ (4) ${}^{100}C_{15}$

Ans. (4)

Sol. $\Rightarrow (1-x)^{101}(x^2+x+1)^{100}$
 $\Rightarrow (1-x)^{100}(x^2+x+1)^{100}(1-x)$
 $\Rightarrow (1-x^3)^{100}(1-x)$
 $\Rightarrow (1-x)({}^{100}C_0 - {}^{100}C_1x^3 + {}^{100}C_2x^6 + \dots + {}^{100}C_{84}x^{252} - {}^{100}C_{85}x^{255} + {}^{100}C_{86}x^{256} + \dots)$
 $\Rightarrow {}^{100}C_{85}x^{256}$

so, the coefficient of x^{256} is ${}^{100}C_{85}$

4. Let a be a positive real number such that $\int_0^a e^{x-[x]} dx = 10e - 9$ where $[x]$ is the greatest integer less than or equal to x. Then a is equal to :

माना a एक धनात्मक वास्तविक संख्या है जिसके लिए $\int_0^a e^{x-[x]} dx = 10e - 9$ है, जहाँ $[x]$ महत्तम पूर्णांक $\leq x$ है। तो a बराबर है :

- (1) $10 + \log_e(1+e)$ (2) $10 - \log_e(1+e)$ (3) $10 + \log_e 2$ (4) $10 + \log_e 3$

Ans. (3)

Sol. Let $a = 10 + K$, $0 \leq K < 1$

$$\int_0^a e^{x-[x]} dx = 10e - 9$$

$$\int_0^{10} e^{x-[x]} dx + \int_{10}^{10+K} e^{x-[x]} dx = 10e - 10 + 1 \Rightarrow \int_{10}^{10+K} e^{x-[x]} dx = 1 \Rightarrow \int_0^K e^{x-[x]} dx = 1$$

$$e^K - 1 = 1$$

$$K = \ln 2$$

$$\text{so, } a = 10 + \ln 2$$

5. Let $y = y(x)$ be the solution of the differential equation

$x \tan\left(\frac{y}{x}\right) dy = \left(y \tan\left(\frac{y}{x}\right) - x\right) dx$, $-1 \leq x \leq 1$, $y\left(\frac{1}{2}\right) = \frac{\pi}{6}$. Then the area of the region bounded by the

curves $x = 0$, $x = \frac{1}{\sqrt{2}}$ and $y = y(x)$ in the upper half plane is :

माना अवकल समीकरण

$$x \tan\left(\frac{y}{x}\right) dy = \left(y \tan\left(\frac{y}{x}\right) - x\right) dx, \quad -1 \leq x \leq 1, \quad y\left(\frac{1}{2}\right) = \frac{\pi}{6}.$$

का हल $y = y(x)$ है। तो वक्रों $x = 0$, $x = \frac{1}{\sqrt{2}}$ तथा $y = y(x)$ द्वारा ऊपरी आधे निर्देशांक तल में घिरे क्षेत्र का क्षेत्रफल है :

(1) $\frac{1}{6}(\pi - 1)$

(2) $\frac{1}{12}(\pi - 3)$

(3) $\frac{1}{8}(\pi - 1)$

(4) $\frac{1}{4}(\pi - 2)$

Ans. (3)

Sol. $x \tan\left(\frac{y}{x}\right) dy = \left(y \tan\left(\frac{y}{x}\right) - x\right) dx$

$$\Rightarrow \tan\left(\frac{y}{x}\right) dy = \left(\frac{y}{x} \tan\left(\frac{y}{x}\right) - 1\right) dx$$

let $\left(\frac{y}{x}\right) = t$

$$\Rightarrow \frac{dy}{dx} = t + x \frac{dt}{dx}$$

$$\Rightarrow t + x \frac{dt}{dx} = \frac{t \tan t - 1}{\tan t}$$

$$\Rightarrow t + \frac{x dt}{dx} = t - \cot t$$

$$\Rightarrow \int \tan t dt = -\int \frac{dx}{x} \Rightarrow \log|\sec t| = -\ln|x| + \ln c$$

$$\Rightarrow \ln\left|\sec\left(\frac{y}{x}\right)\right| = \ln\left|\frac{c}{x}\right|$$

When $x > 0$, $y > 0$ then

$$\sec\left(\frac{y}{x}\right) = \frac{c}{x}$$

$$y\left(\frac{1}{2}\right) = \frac{\pi}{6}$$

$$\sec\left(\frac{\pi}{3}\right) = \frac{c}{\frac{1}{2}}$$

$$c = 1$$

$$\sec\left(\frac{y}{x}\right) = \frac{1}{x}$$

$$\frac{y}{x} = \sec^{-1}\left(\frac{1}{x}\right)$$

$$y = x \sec^{-1}\left(\frac{1}{x}\right)$$

$$\text{Area} = \int_0^{\frac{1}{\sqrt{2}}} x \sec^{-1}\left(\frac{1}{x}\right) dx = \int_0^{\frac{1}{\sqrt{2}}} x \cos^{-1} x dx$$

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7. The number of real roots of the equation $\tan^{-1} \sqrt{x(x+1)} + \sin^{-1} \sqrt{x^2+x+1} = \frac{\pi}{4}$ is :

समीकरण $\tan^{-1} \sqrt{x(x+1)} + \sin^{-1} \sqrt{x^2+x+1} = \frac{\pi}{4}$ के वास्तविक मूलों की संख्या है :

- (1) 1 (2) 4 (3) 0 (4) 2

Ans. (3)

Sol. $\tan^{-1} \sqrt{x(x+1)} + \sin^{-1} \sqrt{x^2+x+1} = \frac{\pi}{4}$

as $x^2+x \geq 0 \Rightarrow x^2+x+1 \geq 1$

But $x^2+x+1 \leq 1$

So, $x^2+x=0$

$x=0, -1$

$x=0, -1$ does not satisfies the original equation

\therefore no solution

8. Let a function $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined as $f(x) = \begin{cases} \sin x - e^x ; & \text{if } x \leq 0 \\ a + [-x] ; & \text{if } 0 < x < 1 \\ 2x - b ; & \text{if } x \geq 1 \end{cases}$ where $[x]$ is the greatest integer less

than or equal to x . If f is continuous on \mathbb{R} , then $(a+b)$ is equal to :

माना एक फलन $f : \mathbb{R} \rightarrow \mathbb{R}$

$f(x) = \begin{cases} \sin x - e^x & \text{यदि } x \leq 0 \text{ है} \\ a + [-x] & \text{यदि } 0 < x < 1 \text{ है} \\ 2x - b & \text{यदि } x \geq 1 \text{ है,} \end{cases}$ द्वारा परिभाषित है, जहाँ $[x]$ महत्तम पूर्णांक $\leq x$ है। यदि फलन f , \mathbb{R} पर संतत है,

तो $(a+b)$ बराबर है :

- (1) 4 (2) 5 (3) 3 (4) 2

Ans. (3)

Sol. Since $f(x)$ is continuous at $x=0$

So $\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x) = f(0)$

$-1 = a - 1 = -1 \Rightarrow a = 0$

Since $f(x)$ is continuous at $x=1$

So $\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x) = f(1)$

$a - 1 = 2 - b = 2 - b$

$\Rightarrow a = 0$, so $0 - 1 = 2 - b$

$\Rightarrow -3 = -b$

$\Rightarrow b = 3$

So the value of $a+b = 3$

9. The mean of 6 distinct observations is 6.5 and their variance is 10.25. If 4 out of 6 observations are 2, 4, 5 and 7, then the remaining two observations are :

6 भिन्न प्रेक्षणों का माध्य 6.5 है तथा उनका प्रसरण 10.25 है। यदि 6 प्रेक्षणों में से 4 प्रेक्षण 2, 4, 5 तथा 7 हैं, तो शेष दो प्रेक्षण हैं :

- (1) 3, 18 (2) 10, 11 (3) 1, 20 (4) 8, 13

Ans. (2)

Sol. Let two number x and y according to question

$$18 + x + y = 39$$

$$x + y = 21 \quad \dots\dots\dots(1)$$

$$10.25 = \frac{\sum x_i^2}{n} - (\bar{x})^2$$

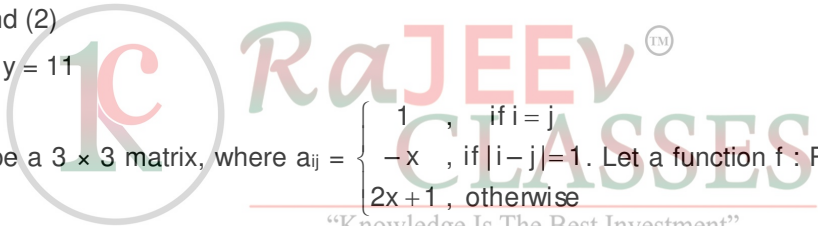
$$10.25 = \frac{x^2 + y^2 + 4 + 16 + 25 + 49}{6} - (6.5)^2$$

$$10.25 = \frac{x^2 + y^2 + 94}{6} - (6.5)^2$$

$$\Rightarrow x^2 + y^2 = 221 \quad \dots\dots\dots(2)$$

solving (1) and (2)

So, x = 10 or y = 11



10. Let $A = [a_{ij}]$ be a 3×3 matrix, where $a_{ij} = \begin{cases} 1, & \text{if } i = j \\ -x, & \text{if } |i - j| = 1 \\ 2x + 1, & \text{otherwise} \end{cases}$. Let a function $f : R \rightarrow R$ be defined as

$f(x) = \det(A)$. Then the sum of maximum and minimum values of f on R is equal to :

माना $A = [a_{ij}]$ एक 3×3 का आव्यूह है, जहाँ $a_{ij} = \begin{cases} 1, & \text{यदि } i = j \text{ है} \\ -x, & \text{यदि } |i - j| = 1 \text{ है} \\ 2x + 1, & \text{अन्यथा,} \end{cases}$

माना एक फलन $f : R \rightarrow R, f(x) = \det(A)$ द्वारा परिभाषित है। तो R पर f के अधिकतम तथा निम्नतम मानों का योगफल बराबर है :

- (1) $\frac{20}{27}$ (2) $-\frac{20}{27}$ (3) $-\frac{88}{27}$ (4) $\frac{88}{27}$

Ans. (3)

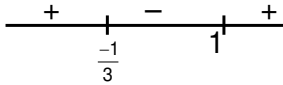
Sol. $|A| = \begin{vmatrix} 1 & -x & 2x+1 \\ -x & 1 & -x \\ 2x+1 & -x & 1 \end{vmatrix} = 1 + x^2(2x+1) + x^2(2x+1) - (2x+1)^2 - x^2 - x^2$

$$\Rightarrow f(x) = 4x^3 - 4x^2 - 4x$$

$$\Rightarrow f'(x) = 12x^2 - 8x - 4$$

$$= 4(3x^2 - 2x - 1)$$

$$= 4(x-1)(3x+1)$$



$\Rightarrow f(x)$ is maximum at $x = \frac{-1}{3}$ and minimum at $x = 1$

\therefore maximum value = $\frac{20}{27}$ and minimum value = -4

$$\therefore \text{sum} = \frac{20}{27} - 4 = -\frac{88}{27}$$

11. The Boolean expression $(p \wedge \sim q) \Rightarrow (q \vee \sim p)$ is equivalent to :

बूलीय व्यंजक $(p \wedge \sim q) \Rightarrow (q \vee \sim p)$ निम्न में से किसके तुल्य है ?

(1) $p \Rightarrow \sim q$

(2) $\sim q \Rightarrow p$

(3) $q \Rightarrow p$

(4) $p \Rightarrow q$

Ans. (4)

Sol. $(p \wedge \sim q) \Rightarrow (q \vee \sim p)$

$$\Rightarrow (\sim p \vee q) \vee (q \vee \sim p)$$

$$\Rightarrow \sim p \vee q$$

$$= p \Rightarrow q$$



12. Let $[x]$ denote the greatest integer $\leq x$, where $x \in \mathbb{R}$. If the domain of the real valued function

$$f(x) = \sqrt{\frac{[x]-2}{[x]-3}}$$
 is $(-\infty, a) \cup [b, c] \cup [4, \infty)$, $a < b < c$, then the value of $a + b + c$ is :

माना $[x]$ महत्तम पूर्णांक $\leq x$ है, जहाँ $x \in \mathbb{R}$ है। यदि वास्तविक मान फलन $f(x) = \sqrt{\frac{[x]-2}{[x]-3}}$ का प्रांत

$(-\infty, a) \cup [b, c] \cup [4, \infty)$, $a < b < c$, है, तो $a + b + c$ का मान है :

(1) 1

(2) -2

(3) 8

(4) -3

Ans. (2)

Sol.

$$\frac{[x]-2}{[x]-3} \geq 0 \cap [x]-3 \neq 0$$

$$\text{Let } t \in [x], t > 0$$

$$\Rightarrow \frac{t-2}{t-3} \geq 0$$

$$\Rightarrow t \in (-\infty, 2] \cup (3, \infty) \cap t > 0$$

$$\Rightarrow [x] \in [0, 2] \cup (3, \infty)$$

$$\Rightarrow [x] \in (-\infty, -3) \cup [-2, 2] \cup (3, \infty)$$

$$\Rightarrow [x] \in (-\infty, -3) \cup [-2, 3] \cup (4, \infty)$$

So $a = -3, b = -2, c = 3$
 So $a + b + c = -2$

13. Let $y = y(x)$ be the solution of the differential equation $e^x \sqrt{1-y^2} dx + \left(\frac{y}{x}\right) dy = 0, y(1) = -1$. Then the value of $(y(3))^2$ is equal to :

माना अवकल समीकरण $e^x \sqrt{1-y^2} dx + \left(\frac{y}{x}\right) dy = 0, y(1) = -1$ का हल $y = y(x)$ है। तो $(y(3))^2$ का मान बराबर है :

- (1) $1 + 4e^6$ (2) $1 + 4e^3$ (3) $1 - 4e^3$ (4) $1 - 4e^6$

Ans. (4)

Sol. $e^x \sqrt{1-y^2} dx = -\frac{y}{x} dy$

$$\int x e^x dx = \int \frac{-y}{\sqrt{1-y^2}} dy$$

Put $1-y^2 = t^2$

$$-2y dy = 2t dt$$

$$\Rightarrow x e^x - e^x = \int \frac{tdt}{t}$$

$$\Rightarrow x e^x - e^x = \sqrt{1-y^2} + c$$

Given $y(1) = -1$

$$\Rightarrow 0 = 0 + c \Rightarrow c = 0$$

$$x e^x - e^x = \sqrt{1-y^2}$$

Put $x = 3$

$$(3-1) e^3 = \sqrt{1-y^2}$$

$$4e^6 = 1 - y^2$$

$$(y(3))^2 = 1 - 4e^6$$

14. If α and β are the distinct roots of the equation $x^2 + (3)^{1/4} x + 3^{1/2} = 0$, then the value of $\alpha^{96}(\alpha^{12} - 1) \beta^{96}(\beta^{12} - 1)$ is equal to :

यदि α तथा β , समीकरण $x^2 + (3)^{1/4} x + 3^{1/2} = 0$ के दो भिन्न मूल हैं, तो $\alpha^{96}(\alpha^{12} - 1) + \beta^{96}(\beta^{12} - 1)$ का मान बराबर है :

- (1) 56×3^{24} (2) 52×3^{24} (3) 56×3^{25} (4) 28×3^{25}

Ans. (2)



Sol. $x^2 + \sqrt{3} = -3^{\frac{1}{4}}x$

$$\Rightarrow x^4 + 2\sqrt{3}x^2 + 3 = \sqrt{3}x^2$$

$$\Rightarrow x^4 + \sqrt{3}x^2 + 3 = 0$$

$$\Rightarrow x^8 + 6x^4 + 9 = 3x^4$$

$$\Rightarrow x^8 + 3x^4 + 9 = 0$$

$$\Rightarrow \alpha^8 = -9 - 3\alpha^4$$

$$\Rightarrow \alpha^{12} = -9\alpha^4 - 3\alpha^8 = -9\alpha^4 - 3(-9 - 3\alpha^4) = 27$$

Similarly $\beta^{12} = 27$

$$\Rightarrow \alpha^{96}(\alpha^{12} - 1) + \beta^{96}(\beta^{12} - 1) = (27)^8 \cdot 26 + (27)^8 \cdot 26 = 52 \cdot (27)^8 = 52 \cdot 3^{24}$$

15. If in a triangle ABC, $AB = 5$ units, $\angle B = \cos^{-1}\left(\frac{3}{5}\right)$ and radius of circumcircle of $\triangle ABC$ is 5 units, then the area (in sq. units) of $\triangle ABC$ is :

यदि एक त्रिभुज ABC में $AB = 5$ इकाई, $\angle B = \cos^{-1}\left(\frac{3}{5}\right)$ तथा $\triangle ABC$ के परिवृत्त की त्रिज्या 5 इकाई है, तो त्रिभुज

$\triangle ABC$ का क्षेत्रफल (वर्ग इकाइयों में) है :

(1) $4 + 2\sqrt{3}$

(2) $8 + 2\sqrt{2}$

(3) $6 + 8\sqrt{3}$

(4) $10 + 6\sqrt{2}$

Ans. (3)

Sol. $\cos B = \frac{3}{5} \Rightarrow \sin B = \frac{4}{5}$, $R = 5$

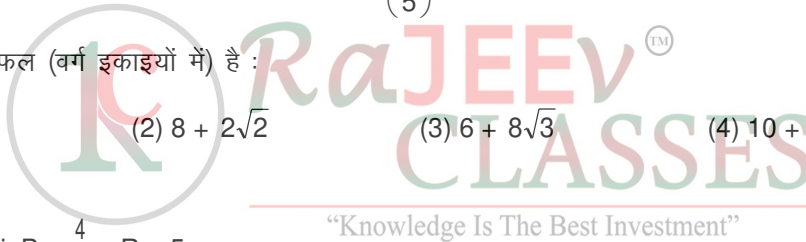
$$\Rightarrow \frac{b}{2R} = \frac{4}{5} \Rightarrow b = 8, c = 5$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{3}{5} \Rightarrow \frac{a^2 + 25 - 64}{2a(5)} = \frac{3}{5}$$

$$a^2 - 39 = 6a \Rightarrow a^2 - 6a - 39 = 0$$

$$\Rightarrow a = \frac{6 + 8\sqrt{3}}{2} \Rightarrow a = 3 + 4\sqrt{3}$$

$$\Delta = \frac{abc}{4R} = \frac{(3 + 4\sqrt{3})(8)(5)}{4(5)} = 6 + 8\sqrt{3}$$



16. Words with or without meaning are to be formed using all the letters of the word EXAMINATION. The probability that the letter M appears at the fourth position in any such word is :
- शब्द EXAMINATION के सभी अक्षरों का उपयोग कर अर्थपूर्ण या अर्थहीन शब्द बनाये जाने हैं। ऐसे किसी शब्द में M अक्षर के चौथे स्थान पर होने की प्रायिकता है :

(1) $\frac{1}{11}$

(2) $\frac{1}{66}$

(3) $\frac{2}{11}$

(4) $\frac{1}{9}$

Ans. (1)

Sol. EXAMINATION

E → 1 $n(S) = \frac{11!}{2! 2! 2!}$

X → 1 $n(E) = \frac{10!}{2! 2! 2!}$

A → 2 $P(E) = \frac{n(E)}{n(s)} = \frac{1}{11}$

M → 1

O → 1

T → 1

N → 2

I → 2

17. The probability of selecting integers $a \in [-5, 30]$ such that $x^2 + 2(a + 4)x - 5a + 64 > 0$ for all $x \in \mathbb{R}$, is :
- पूर्णांक $a \in [-5, 30]$ चुनने की प्रायिकता, जबकि $x^2 + 2(a + 4)x - 5a + 64 > 0$, $x \in \mathbb{R}$, है, है :

(1) $\frac{7}{36}$

(2) $\frac{2}{9}$

(3) $\frac{1}{6}$

(4) $\frac{1}{4}$

Ans. (2)

Sol. $x^2 + 2(a+4)x - (5a - 64) > 0$

$D < 0$

$\therefore 4(a+4)^2 - 4(5a - 64) < 0$

$\Rightarrow (a+4)^2 + (5a - 64) < 0$

$\Rightarrow a^2 + 13a - 48 < 0$

$a = \frac{-13 \pm \sqrt{169 + 192}}{2} = -16, 3$

So, $a \in (-16, 3)$

So, $a = -5, -4, -3, -2, -1, 0, 1, 2$

\therefore Required Probability = $\frac{8}{36} = \frac{2}{9}$

18. Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$. If \vec{c} is a vector such that $\vec{a} \cdot \vec{c} = |\vec{c}|$, $|\vec{c} - \vec{a}| = 2\sqrt{2}$ and the angle between $(\vec{a} \times \vec{b})$ and \vec{c} is $\frac{\pi}{6}$, then the value of $|(\vec{a} \times \vec{b}) \times \vec{c}|$ is :

माना $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$ तथा $\vec{b} = \hat{i} + \hat{j}$ हैं। यदि एक सदिश \vec{c} के लिए $\vec{a} \cdot \vec{c} = |\vec{c}|$, $|\vec{c} - \vec{a}| = 2\sqrt{2}$ हैं तथा $(\vec{a} \times \vec{b})$ और \vec{c} के बीच का कोण $\frac{\pi}{6}$ है, तो $|(\vec{a} \times \vec{b}) \times \vec{c}|$ का मान है :

- (1) $\frac{2}{3}$ (2) 3 (3) $\frac{3}{2}$ (4) 4

Ans. (3)

Sol.
$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 1 & 1 & 0 \end{vmatrix}$$

$$= \hat{i}(2) - \hat{j}(2) + \hat{k}(1)$$

$$= 2\hat{i} - 2\hat{j} + \hat{k}$$

$$|\vec{a} \times \vec{b}| = 3$$

$$|\vec{c} - \vec{a}|^2 = 8$$

$$|\vec{c}|^2 + |\vec{a}|^2 - 2\vec{a} \cdot \vec{c} = 8$$

$$|\vec{c}|^2 + 9 - 2|\vec{c}| = 8$$

$$|\vec{c}|^2 - 2|\vec{c}| + 1 = 0$$

$$|\vec{c}| = 1$$

$$|(\vec{a} \times \vec{b}) \times \vec{c}| = |\vec{a} \times \vec{b}| |\vec{c}| \sin \frac{\pi}{6}$$

$$3 \times 1 \times \frac{1}{2} = \frac{3}{2}$$

19. Let $A = \begin{bmatrix} 2 & 3 \\ a & 0 \end{bmatrix}$, $a \in \mathbb{R}$ be written as $P + Q$ where P is a symmetric matrix and Q is skew symmetric matrix.

If $\det(Q) = 9$, then the modulus of the sum of all possible values of determinant of P is equal to :

माना $A = \begin{bmatrix} 2 & 3 \\ a & 0 \end{bmatrix}$, $a \in \mathbb{R}$ को $P + Q$ के रूप में लिखा गया है, जहाँ P एक सममित आव्यूह है तथा Q एक विषम सममित

आव्यूह है। यदि $\det(Q) = 9$ है, तो $\det(P)$ के सभी संभव मानों के योगफल का मापांक बराबर है :

- (1) 45 (2) 24 (3) 18 (4) 36



Ans. (4)

Sol. $\because A = \frac{1}{2} (A+A^T) + \frac{1}{2} (A-A^T)$

Where $A+A^T$ is symmetric and $(A-A^T)$ is skew symmetric

$$\Rightarrow P = \frac{1}{2} (A+A^T) \text{ and } Q = \frac{1}{2} (A-A^T)$$

$$\Rightarrow Q = \frac{1}{2} \begin{bmatrix} 0 & 3-a \\ a-3 & 0 \end{bmatrix}$$

$$\Rightarrow \det(Q) = \frac{1}{4} (a-3)^2 = 9$$

$$\Rightarrow (a-3)^2 = 36$$

$$\Rightarrow a = 9 \text{ or } -3$$

Now $P = \frac{1}{2} \begin{bmatrix} 4 & 3+a \\ a+3 & 0 \end{bmatrix}$

$$\Rightarrow \det(P) = -\frac{1}{4} (a+3)^2 = -36 \text{ or } 0$$

$$\Rightarrow \text{sum of all possible values of } \det(P) = 36$$

20 If z and ω are two complex numbers such that $|z\omega| = 1$, $\arg(z) - \arg(\omega) = \frac{3\pi}{2}$, then $\arg\left(\frac{1-2\bar{z}\omega}{1+3\bar{z}\omega}\right)$, is :

(Here $\arg(z)$ denotes the principal argument of complex number z)

यदि z तथा ω दो सम्मिश्र संख्याएँ हैं जिनके लिए $|z\omega| = 1$, $\arg(z) - \arg(\omega) = \frac{3\pi}{2}$ हैं, तो $\arg\left(\frac{1-2\bar{z}\omega}{1+3\bar{z}\omega}\right)$ बराबर है :

(जहाँ $\arg(z)$ सम्मिश्र संख्या z के मुख्य कोणांक को दर्शाता है)

(1) $-\frac{\pi}{4}$

(2) $\frac{3\pi}{4}$

(3) $-\frac{3\pi}{4}$

(4) $\frac{\pi}{4}$

Ans. (3)

Sol. $|z\omega| = 1$ and $\arg(z) - \arg(\omega) = \frac{3\pi}{2}$

$$|z||\omega| = 1 \text{ and } \arg(z) = \arg(\omega) + \frac{3\pi}{2}$$

Let $w = re^{i\theta}$

$$\Rightarrow z = \frac{1}{r} e^{i\left(\frac{3\pi}{2} + \theta\right)}$$

$$\Rightarrow \frac{1-2\bar{z}\omega}{1+3\bar{z}\omega} = \frac{1-2re^{i\theta} \frac{1}{r} e^{i\left(-\frac{3\pi}{2}-\theta\right)}}{1+3r e^{i\theta} \frac{1}{r} e^{i\left(-\frac{3\pi}{2}-\theta\right)}}$$

$$= \frac{1-2e^{-i3\pi/2}}{1+3e^{-i3\pi/2}} = \frac{1-2i}{1+3i}$$

$$= \arg\left(\frac{1-2\bar{z}\omega}{1+3\bar{z}\omega}\right) = \arg\left(\frac{1-2i}{1+3i}\right) = \tan^{-1}(-2) - \tan^{-1}(3) = -\frac{3\pi}{4}$$

“Knowledge Is The Best Investment”

INTEGER TYPE

1. Let $A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$ and $B = 7A^{20} - 20A^7 + 2I$, where I is an identity matrix of order 3×3 . If $B = [b_{ij}]$,

then b_{13} is equal to –

माना $A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$ तथा $B = 7A^{20} - 20A^7 + 2I$ हैं, जहाँ I , 3×3 कोटि का तत्समक आव्यूह है। यदि

$B = [b_{ij}]$, तो b_{13} बराबर है –

Ans. (910)

Sol. Let $A = I + C$

Where $C = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix}$

$C^2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$C^3 = O$

So

$A^2 = (I + C)^2 = I + 2C + C^2$

$A^3 = A^2 \cdot A = I + 3C + 3C^2$

$A^4 = I + 4C + 6C^2$

$A^5 = I + 5C + 10C^2$

So, $A^n = I + nC + \frac{n(n-1)}{2}C^2$

$A^{20} = I + 20C + 190C^2$

$A^7 = I + 7C + 21C^2$

$B = 7A^{20} - 20A^7 + 2I$

$\Rightarrow B = -11I + 910C^2 = \begin{bmatrix} -11 & 0 & 910 \\ 0 & -11 & 0 \\ 0 & 0 & -11 \end{bmatrix}$

$\therefore b_{13} = 910$

2. Let $\vec{a}, \vec{b}, \vec{c}$ be three mutually perpendicular vectors of the same magnitude and equally inclined at an angle θ with the vector $\vec{a} + \vec{b} + \vec{c}$. Then $36\cos^2 2\theta$ is equal to –

माना बराबर परिमाण के तीन सदिश $\vec{a}, \vec{b}, \vec{c}$ परस्पर लम्बवत् हैं तथा सदिश $\vec{a} + \vec{b} + \vec{c}$ के साथ एक समान कोण θ बनाते हैं। तो $36\cos^2 2\theta$ बराबर है—

Ans. (4)

Sol. $|\vec{a} + \vec{b} + \vec{c}|^2 = (\vec{a} + \vec{b} + \vec{c}) \cdot (\vec{a} + \vec{b} + \vec{c}) = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 = 3$

$$\Rightarrow |\vec{a} + \vec{b} + \vec{c}| = \sqrt{3}$$

Now $\vec{a} \cdot (\vec{a} + \vec{b} + \vec{c}) = |\vec{a}| |\vec{a} + \vec{b} + \vec{c}| \cos\theta$

$$\Rightarrow \cos\theta = \frac{1}{\sqrt{3}} \Rightarrow \cos 2\theta = 2\cos^2\theta - 1$$

$$\Rightarrow \cos 2\theta = -\frac{1}{3} \Rightarrow \cos^2 2\theta = \frac{1}{9} \Rightarrow 36\cos^2 2\theta = 4$$

3. There are 15 players in a cricket team, out of which 6 are bowlers, 7 are batsmen and 2 are wicketkeepers. Then number of ways, a team of 11 players be selected from them so as to include at least 4 bowlers, 5 batsmen and 1 wicketkeeper, is –

एक क्रिकेट टीम में 15 खिलाड़ी हैं जिनमें 6 गेंदबाज, 7 बल्लेबाज तथा 2 विकेट कीपर हैं। इनमें से 11 खिलाड़ियों की एक टीम, जिसमें कम से कम 4 गेंदबाज, 5 बल्लेबाज तथा 1 विकेट कीपर हो, चुनने के तरीकों की संख्या है—

Ans. (777)

Sol. **Case-I :** Team consist 5 Batsman , 5 Bollwers

and 1 wicket keeper then number of ways.

$$= {}^6C_5 \times {}^7C_5 \times {}^2C_1 = 6 \times 21 \times 2 = 252$$

Case - II 4 Batsmen, 6 bollwers and 1 wicket keeper

$$= {}^6C_4 \times {}^7C_6 \times {}^2C_1 = 15 \times 7 \times 2 = 210$$

Case-III 4 Batsmen, 5 Bollwers and 2 wicket keepers

$${}^6C_4 \times {}^7C_5 \times {}^2C_2 = 15 \times 21 \times 1 = 315$$

$$\text{Total } 252 + 210 + 315 = 777$$

4. If the shortest distance between the lines $\vec{r}_1 = \alpha\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - 2\hat{j} + 2\hat{k})$, $\lambda \in \mathbb{R}$, $\alpha > 0$ and

$$\vec{r}_2 = -4\hat{i} - \hat{k} + \mu(3\hat{i} - 2\hat{j} - 2\hat{k}), \mu \in \mathbb{R} \text{ is } 9, \text{ then } \alpha \text{ is equal to -}$$

यदि रेखाओं $\vec{r}_1 = \alpha\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - 2\hat{j} + 2\hat{k})$, $\lambda \in \mathbb{R}$, $\alpha > 0$ तथा

$$\vec{r}_2 = -4\hat{i} - \hat{k} + \mu(3\hat{i} - 2\hat{j} - 2\hat{k}), \mu \in \mathbb{R} \text{ के मध्य न्यूनतम दूरी } 9 \text{ है, तो } \alpha \text{ बराबर है—}$$

Ans. (6)

Sol. Shortest distance = $\frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|}$

$$\Rightarrow 9 = \frac{|((\alpha + 4)\hat{i} + 2\hat{j} + 3\hat{k}) \cdot (8\hat{i} + 8\hat{j} + 4\hat{k})|}{\sqrt{64 + 64 + 16}} \quad \left| \frac{8(\alpha + 4) + 16 + 12}{12} \right| = 9$$

$$\therefore \alpha = 6$$

5. The number of rational terms in the binomial expansion of $\left(4^{\frac{1}{4}} + 5^{\frac{1}{6}}\right)^{120}$ is –

$\left(4^{\frac{1}{4}} + 5^{\frac{1}{6}}\right)^{120}$ के द्विपद प्रसार में परिमेय पदों की संख्या है–

Ans. (21)

Sol. General term of $\left(2^{\frac{1}{2}} + 5^{\frac{1}{6}}\right)^{120}$ is

$$\text{given by } T_{r+1} = {}^{120}C_r \left(2^{\frac{1}{2}}\right)^{120-r} \left(5^{\frac{1}{6}}\right)^r$$

For integral term, r should be a multiple of 6

i.e. $r \in \{0, 6, 12, 18, \dots, 120\}$

\therefore 21 integral terms are there in the expansion $\left(2^{\frac{1}{2}} + 5^{\frac{1}{6}}\right)^{120}$

6. Let P be a plane passing through the points (1, 0, 1), (1, -2, 1) and (0, 1, -2). Let a vector $\vec{a} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$ be such that \vec{a} is parallel to the plane P, perpendicular to $(\hat{i} + 2\hat{j} + 3\hat{k})$ and $\vec{a} \cdot (\hat{i} + \hat{j} + 2\hat{k}) = 2$, then $(\alpha - \beta + \gamma)^2$ equals –

माना समतल P बिन्दुओं (1, 0, 1), (1, -2, 1) तथा (0, 1, -2) से होकर जाता है। माना एक सदिश $\vec{a} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$, समतल P के समान्तर है, सदिश $(\hat{i} + 2\hat{j} + 3\hat{k})$ के लम्बवत् है, तथा $\vec{a} \cdot (\hat{i} + \hat{j} + 2\hat{k}) = 2$ को संतुष्ट करता है। तो $(\alpha - \beta + \gamma)^2$ बराबर है –

Ans. (81)

Sol. Equation of plane P is

$$\begin{vmatrix} x-1 & y-0 & z-1 \\ 0 & -2 & 0 \\ -1 & 1 & -3 \end{vmatrix} = 0$$

$$\Rightarrow -3(x+1) + (z-1) = 0$$

$$\Rightarrow 3x - z - 2 = 0$$

Normal vector to the plane P is $\vec{n} = 3\hat{i} - \hat{k}$.

Now $\vec{a} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$ is perpendicular to \vec{n}

$$\Rightarrow \vec{a} \cdot \vec{n} = 0$$

$$\Rightarrow 3\alpha - \gamma = 0 \quad \dots\dots(1)$$

also \vec{a} is perpendicular to $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$

$$\Rightarrow \vec{a} \cdot \vec{b} = 0$$

$$\Rightarrow \alpha + 2\beta + 3\gamma = 0 \quad \dots\dots(2)$$

$$\text{and } \vec{a} \cdot (\hat{i} + \hat{j} + 2\hat{k}) = 2 \Rightarrow \alpha + \beta + 2\gamma = 2 \quad \dots\dots(3)$$

by solving (1), (2) and (3)

$$\alpha = 1, \beta = -5, \gamma = 3 \Rightarrow (\alpha - \beta + \gamma)^2 = 81$$

7. Let a, b, c, d be in arithmetic progression with common difference λ . If

$$\begin{vmatrix} x+a-c & x+b & x+a \\ x-1 & x+c & x+b \\ x-b+d & x+d & x+c \end{vmatrix} = 2, \text{ then value of } \lambda^2 \text{ is equal to -}$$

माना a, b, c, d एक समान्तर श्रेणी में हैं जिसका सार्व अन्तर λ है। यदि $\begin{vmatrix} x+a-c & x+b & x+a \\ x-1 & x+c & x+b \\ x-b+d & x+d & x+c \end{vmatrix} = 2$ है, तो λ^2 का मान

बराबर है—

Ans. (1)

Sol.

$$\begin{vmatrix} x+a-c & x+b & x+a \\ x-1 & x+c & x+b \\ x-b+d & x+d & x+c \end{vmatrix} = 2$$

$$\Rightarrow \begin{vmatrix} x-2\lambda & x+b & x+a \\ x-1 & x+c & x+b \\ x+2\lambda & x+d & x+c \end{vmatrix} = 2 \quad (\because c-a = 2\lambda = d-b)$$

$$R_2 \rightarrow R_2 - R_1 \quad \& \quad R_3 \rightarrow R_3 - R_2$$

$$\Rightarrow \begin{vmatrix} x-2\lambda & x+b & x+a \\ -1+2\lambda & \lambda & \lambda \\ 2\lambda+1 & \lambda & \lambda \end{vmatrix} = 2$$

$$R_3 \rightarrow R_3 - R_2$$

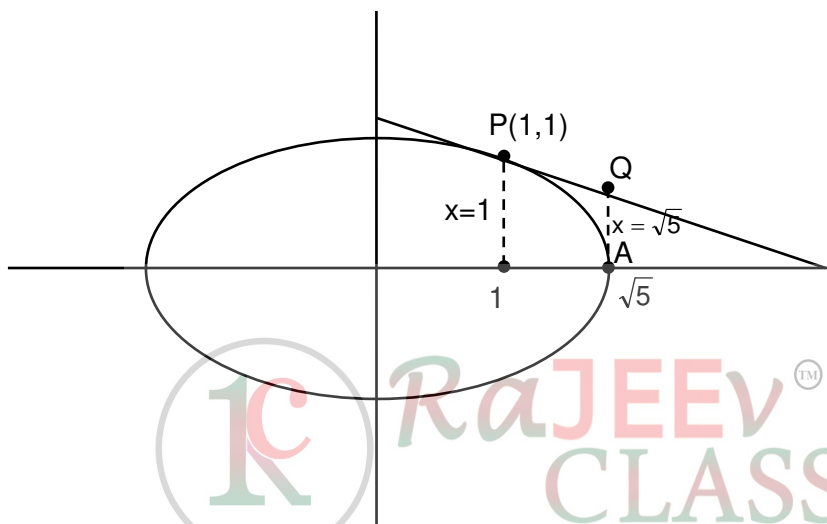
$$\Rightarrow \begin{vmatrix} x-2\lambda & x+b & x+a \\ 2\lambda-1 & \lambda & \lambda \\ 2 & 0 & 0 \end{vmatrix} = 2 \Rightarrow 2\lambda(x+b-x-a) = 2 \Rightarrow \lambda^2 = 1$$



8. Let T be the tangent to the ellipse $E : x^2 + 4y^2 = 5$ at the point $P(1, 1)$. If the area of the region bounded by the tangent T, ellipse E, lines $x = 1$ and $x = \sqrt{5}$ is $\alpha\sqrt{5} + \beta + \gamma \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$, then $|\alpha + \beta + \gamma|$ is equal to
- माना दीर्घवृत्त $E : x^2 + 4y^2 = 5$ के बिन्दु $P(1, 1)$ पर स्पर्श रेखा T है। यदि स्पर्श रेखा T, दीर्घवृत्त E तथा रेखाओं $x = 1$ और $x = \sqrt{5}$ द्वारा घिरे क्षेत्र का क्षेत्रफल $\alpha\sqrt{5} + \beta + \gamma \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$ है, तो $|\alpha + \beta + \gamma|$ बराबर है—

Ans. (1.25)

Sol.



Equation of tangent at $P(1, 1)$ is $x + 4y = 5$.

Now area bounded by the required region

$$\begin{aligned}
 &= \int_1^{\sqrt{5}} \left(\left(\frac{5-x}{4} \right) - \sqrt{\frac{5-x^2}{4}} \right) dx \\
 &= \left(\frac{5}{4}x - \frac{x^2}{8} \right) \Big|_1^{\sqrt{5}} - \frac{1}{2} \left[\frac{1}{2} \times \sqrt{5-x^2} + \frac{5}{2} \sin^{-1} \frac{x}{\sqrt{5}} \right] \Big|_1^{\sqrt{5}} \\
 &= \left(\frac{5\sqrt{5}}{4} - \frac{5}{8} \right) - \left(\frac{5}{4} - \frac{1}{8} \right) - \frac{1}{2} \left[\left(0 + \frac{5\pi}{4} \right) \right] - \left(1 + \frac{5}{2} \sin^{-1} \frac{1}{\sqrt{5}} \right) \\
 &= \frac{5}{4}(\sqrt{5}-1) - \frac{1}{2} - \frac{5\pi}{8} + \frac{1}{2} + \frac{5}{4} \sin^{-1} \frac{1}{\sqrt{5}} \\
 &= \frac{5\sqrt{5}}{4} - \frac{5}{4} - \frac{5}{4} \cos^{-1} \frac{1}{\sqrt{5}} \Rightarrow \alpha = \frac{5}{4}, \beta = -\frac{5}{4} \text{ and } \gamma = -\frac{5}{4} \\
 &\Rightarrow |\alpha + \beta + \gamma| = \frac{5}{4}
 \end{aligned}$$

9. Let $y = mx + c$, $m > 0$ be the focal chord of $y^2 = -64x$, which is tangent to $(x + 10)^2 + y^2 = 4$. Then, the value of $4\sqrt{2}(m + c)$ is equal to –

माना $y = mx + c$, $m > 0$, परवलय $y^2 = -64x$ की नाभीय जीवा है तथा वृत्त $(x + 10)^2 + y^2 = 4$ की स्पर्श रेखा है। तो $4\sqrt{2}(m + c)$ का मान बराबर है –

Ans. (34)

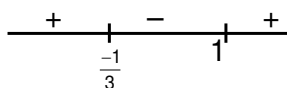
Sol. $|A| = \begin{vmatrix} 1 & -x & 2x+1 \\ -x & 1 & -x \\ 2x+1 & -x & 1 \end{vmatrix} = 1 + x^2(2x+1) + x^2(2x+1) - (2x+1)^2 - x^2 - x^2$

$$\Rightarrow f(x) = 4x^3 - 4x^2 - 4x$$

$$\Rightarrow f'(x) = 12x^2 - 8x - 4$$

$$= 4(3x^2 - 2x - 1)$$

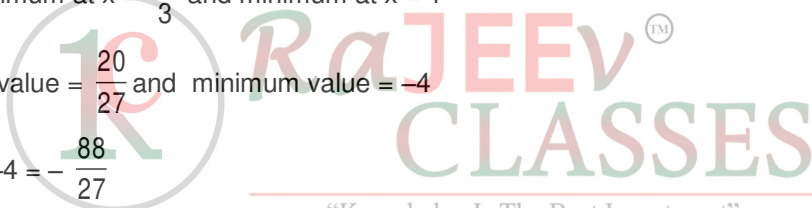
$$= 4(x-1)(3x+1)$$



$\Rightarrow f(x)$ is maximum at $x = -\frac{1}{3}$ and minimum at $x = 1$

\therefore maximum value = $\frac{20}{27}$ and minimum value = -4

$$\therefore \text{sum} = \frac{20}{27} - 4 = -\frac{88}{27}$$



10. If the value $\lim_{x \rightarrow 0} (2 - \cos x \sqrt{\cos 2x})^{\frac{x+2}{x^2}}$ is equal to e^a , then a is equal to

यदि $\lim_{x \rightarrow 0} (2 - \cos x \sqrt{\cos 2x})^{\frac{x+2}{x^2}}$ का मान e^a है, तो a बराबर है–

Ans. (3)

Sol. $\lim_{x \rightarrow 0} (2 - \cos x \sqrt{\cos 2x})^{\frac{x+2}{x^2}} = e^{\lim_{x \rightarrow 0} \frac{(1 - \cos x \sqrt{\cos 2x})(x+2)}{x^2}}$

$$= e^{\lim_{x \rightarrow 0} \frac{(1 - \cos^2 x \cos 2x)(x+2)}{x^2(1 + \cos x \sqrt{\cos 2x})}} = e^{\lim_{x \rightarrow 0} \frac{(1 - (1 - \sin^2 x)(1 - 2\sin^2 x))(x+2)}{x^2(1 + \cos x \sqrt{\cos 2x})}}$$

$$= e^{\lim_{x \rightarrow 0} \frac{(3\sin^2 x - 2\sin^4 x)(x+2)}{x^2(1 + \cos x \sqrt{\cos 2x})}}$$

$$= e^{\lim_{x \rightarrow 0} \frac{\sin^2 x (3 - 2\sin^2 x)(x+2)}{x^2(1 + \cos x \sqrt{\cos 2x})}}$$

$$= e^{\frac{3 \times 2}{1+1}}$$

$$= e^3$$

$\therefore a = 3$