

## Single Choice Type

This section contains **20 Single choice questions**. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which **Only One** is correct.

1. In an electromagnetic wave the electric field vector and magnetic field vector are given as  $\vec{E} = E_0 \hat{i}$  and  $\vec{B} = B_0 \hat{k}$  respectively. The direction of propagation of electromagnetic wave is along

- (1)  $(-\hat{j})$                       (2)  $\hat{j}$                       (3)  $(-\hat{k})$                       (4)  $\hat{k}$

**Ans.** (1)

**Sol.** Direction of EM wave is given by direction of  $\vec{E} \times \vec{B}$ .

$$\text{Unit vector in direction } \vec{E} \times \vec{B} \Rightarrow \frac{\vec{E} \times \vec{B}}{|\vec{E} \times \vec{B}|}$$

$$\Rightarrow \frac{E_0 \hat{i} \times B_0 \hat{k}}{E_0 B_0 \sin 90} = \hat{i} \times \hat{k} = -\hat{j}$$

2. The length of a metal wire is  $l_1$ , when the tension in it is  $T_1$  and is  $l_2$  when the tension is  $T_2$ . The natural length of the wire is :

- (1)  $\sqrt{l_1 l_2}$                       (2)  $\frac{l_1 + l_2}{2}$                       (3)  $\frac{l_1 T_2 - l_2 T_1}{T_2 - T_1}$                       (4)  $\frac{l_1 T_2 + l_2 T_1}{T_2 + T_1}$

**Ans.** (3)

**Sol.** Let initial length of rod be  $l_0$  and Area  $A$ .

$$\text{As } \frac{T}{A} = Y \frac{\Delta \ell}{\ell}$$

$$\text{So, } \frac{T_1}{A} = \frac{Y(l_1 - l_0)}{l_0} ; \frac{T_2}{A} = \frac{Y(l_2 - l_0)}{l_0}$$

$$\text{Dividing } \frac{T_1}{T_2} = \frac{l_1 - l_0}{l_2 - l_0} ; T_1 l_2 - T_1 l_0 = T_2 l_1 - T_2 l_0 ; l_0 = \frac{l_1 T_2 - l_2 T_1}{T_2 - T_1}$$

3. With what speed should a galaxy move outward, with respect to earth so that the sodium-D line at wavelength  $5890 \text{ \AA}$  is observed at  $5896 \text{ \AA}$  ?

- (1) 322 km/sec                      (2) 296 km/sec                      (3) 306 km/sec                      (4) 336 km/sec

**Ans.** (3)

$$\text{Sol. } \frac{v_{\text{rel}}}{C} = \frac{\Delta \lambda}{\lambda}$$

$$v_{\text{rel}} = \frac{6}{5890} \times 3 \times 10^8 \approx 306 \text{ km/s}$$

4. If the Kinetic energy of a moving body becomes four times its initial Kinetic energy, then the percentage change in its momentum will be

- (1) 400%                      (2) 100%                      (3) 300%                      (4) 200%

Ans. (2)

Sol. K.E.  $\Rightarrow K = \frac{P^2}{2m}$

$$P \propto \sqrt{K}$$

$$\frac{P_2}{P_1} = \sqrt{\frac{K_2}{K_1}} \Rightarrow \frac{P_2}{P_1} = \sqrt{\frac{4K}{K}}$$

$$\Rightarrow \frac{P_2}{P_1} = 2$$

$$\Rightarrow \frac{P_2 - P_1}{P_1} \% = \left( \frac{P_2}{P_1} - 1 \right) \times 100 = (2 - 1) \times 100 = 100$$

$$\Rightarrow \frac{\Delta P}{P_1} \% = 100\%$$

5. Two vectors  $\vec{P}$  and  $\vec{Q}$  have equal magnitudes. If the magnitude of  $\vec{P} + \vec{Q}$  is  $n$  times the magnitude of  $\vec{P} - \vec{Q}$ , then angle between  $\vec{P}$  and  $\vec{Q}$  is :

- (1)  $\cos^{-1}\left(\frac{n-1}{n+1}\right)$       (2)  $\sin^{-1}\left(\frac{n-1}{n+1}\right)$       (3)  $\cos^{-1}\left(\frac{n^2-1}{n^2+1}\right)$       (4)  $\sin^{-1}\left(\frac{n^2-1}{n^2+1}\right)$

Ans. (3)

Sol.  $|\vec{P} + \vec{Q}| = n |\vec{P} - \vec{Q}|$

$$|\vec{P}|^2 + |\vec{Q}|^2 + 2|\vec{P}||\vec{Q}|\cos\theta = n^2(|\vec{P}|^2 + |\vec{Q}|^2 - 2|\vec{P}||\vec{Q}|\cos\theta)$$

$$2 + 2\cos\theta = n^2(2 - 2\cos\theta)$$

$$\theta = \cos^{-1}\left(\frac{n^2-1}{n^2+1}\right)$$

6. At an angle of  $30^\circ$  to the magnetic meridian, the apparent dip is  $45^\circ$ . Find the true dip :

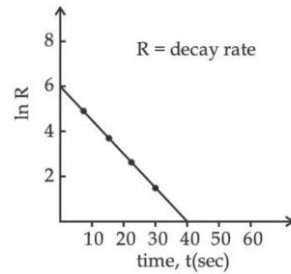
- (1)  $\tan^{-1}\frac{\sqrt{3}}{2}$                       (2)  $\tan^{-1}\frac{1}{\sqrt{3}}$                       (3)  $\tan^{-1}\sqrt{3}$                       (4)  $\tan^{-1}\frac{2}{\sqrt{3}}$

Ans. (1)

Sol.  $\tan 45^\circ = \frac{B\sin\phi}{B\cos\phi\cos 30^\circ}$

$$\tan\phi = \frac{\sqrt{3}}{2}$$

7. For a certain radioactive process the graph between  $\ln R$  and  $t(\text{sec})$  is obtained as shown in the figure. Then the value of half-life for the unknown radioactive material is approximately :



- (1) 4.62 sec                      (2) 2.62 sec                      (3) 9.15 sec                      (4) 6.93 sec

**Ans.** (1)

**Sol.**  $R = R_0 e^{-\lambda t}$

$$\ln R = \ln R_0 - \lambda t$$

$$\text{slope} = -\lambda = \frac{-6}{40}; \quad \lambda = \frac{3}{20}$$

$$t_{1/2} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{3} \times 20 = \frac{20}{3} \ln 2 = 4.62 \text{ sec}$$

8. The magnetic susceptibility of a material of a rod is 499. Permeability in vacuum is  $4\pi \times 10^{-7} \text{ H/m}$ . Absolute permeability of the material of the rod is :

- (1)  $\pi \times 10^{-4} \text{ H/m}$                       (2)  $3\pi \times 10^{-4} \text{ H/m}$                       (3)  $4\pi \times 10^{-4} \text{ H/m}$                       (4)  $2\pi \times 10^{-4} \text{ H/m}$

**Ans.** (4)

**Sol.**  $\mu_r = 1 + \chi$

$$= 1 + 499 = 500$$

$$\text{Absolute permeability} = \mu_0 \times \mu_r$$

$$500 \times 4\pi \times 10^{-7} = 2\pi \times 10^{-4} \text{ H/m}$$

9. A body at rest is moved along a horizontal straight line by a machine delivering a constant power. The distance moved by the body in time 't' is proportional to :

- (1)  $t^{\frac{1}{2}}$                       (2)  $t^{\frac{1}{4}}$                       (3)  $t^{\frac{3}{2}}$                       (4)  $t^{\frac{3}{4}}$

**Ans.** (3)

**Sol.** Energy supply =  $Pt$

in t sec

$$Pt = \frac{1}{2}mv^2; \quad v \propto \sqrt{t}; \quad \frac{dS}{dt} = C\sqrt{t}$$

$$\int_0^S dS = C \int_0^t t^{1/2} dt \Rightarrow S = \frac{2Ct^{3/2}}{3}$$

$$S \propto t^{3/2}$$

10. The correct relation between the degrees of freedom  $f$  and the ratio of specific heat  $\gamma$  is :

(1)  $f = \frac{\gamma+1}{2}$

(2)  $f = \frac{1}{\gamma+1}$

(3)  $f = \frac{2}{\gamma+1}$

(4)  $f = \frac{2}{\gamma-1}$

Ans. (4)

Sol.  $C_V = \frac{fR}{2}$

$\Rightarrow C_P = \left(\frac{f}{2} + 1\right)R$

$\Rightarrow \gamma = \frac{C_P}{C_V} = 1 + \frac{2}{f}$

$\gamma = 1 + \frac{2}{f}$

$f = \frac{2}{\gamma-1}$

11. Consider a binary star system of star A and star B with masses  $m_A$  and  $m_B$  revolving in a circular orbit of radii  $r_A$  and  $r_B$ , respectively. If  $T_A$  and  $T_B$  are the time period of star A and star B, respectively, then ;

(1)  $T_A = T_B$

(2)  $T_A > T_B$  (if  $r_A > r_B$ )

(3)  $\frac{T_A}{T_B} = \left(\frac{r_A}{r_B}\right)^{\frac{3}{2}}$

(4)  $T_A > T_B$  (if  $m_A > m_B$ )

Ans. (1)

Sol. Both the planets will move due to mutual gravitational interaction, thus will have same angular velocity. This will result in same time period.

12. An electron having de-Broglie wavelength  $\lambda$  is incident on a target in a X-ray tube. Cut-off wavelength of emitted X-ray is :

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(1)  $\frac{2m^2c^2\lambda^2}{h^2}$

(2)  $\frac{hc}{mc}$

(3) 0

(4)  $\frac{2mc\lambda^2}{h}$

Ans. (4)

Sol. De-broglie wavelength

$\lambda = \frac{h}{P} \Rightarrow P = \frac{h}{\lambda}$

$\therefore$  Kinetic energy of electron  $\Rightarrow E = \frac{P^2}{2m} = \frac{h^2}{2m\lambda^2}$

For cut-off wavelength of emitted X-Ray

$E = \frac{hc}{\lambda}$

$\Rightarrow \frac{h^2}{2m\lambda^2} = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{2mc\lambda^2}{h}$

13. A boy reaches the airport and finds that the escalator is not working. He walks up the stationary escalator in time  $t_1$ . If he remains stationary on a moving escalator then the escalator takes him up in time  $t_2$ . The time taken by him to walk up on the moving escalator will be :

(1)  $t_2 - t_1$                       (2)  $\frac{t_1 t_2}{t_2 - t_1}$                       (3)  $\frac{t_1 t_2}{t_2 + t_1}$                       (4)  $\frac{t_1 + t_2}{2}$

**Ans.** (3)

**Sol.** Suppose length of escalator =  $L$

Speed of man w.r.t. escalator =  $\frac{L}{t_1}$

Speed of escalator =  $\frac{L}{t_2}$

Speed of man w.r.t. ground when escalator is moving =  $\frac{L}{t_1} + \frac{L}{t_2}$

Time taken by the man to walk on the moving escalator =  $\frac{L}{\frac{L}{t_1} + \frac{L}{t_2}} = \frac{t_1 t_2}{t_1 + t_2}$

14. A satellite is launched into a circular orbit of radius  $R$  around earth, while a second satellite is launched into a circular orbit of radius  $1.02R$ . The percentage difference in the time periods of the two satellites is

(1) 1.5                      (2) 0.7                      (3) 3.0                      (4) 2.0

**Ans.** (3)

**Sol.** As  $T \propto R^{3/2}$

$T_1 = kR^{3/2}$

$\frac{\Delta T}{T} = \frac{3}{2} \times \frac{\Delta R}{R} = 3\%$



15. Two small drops of mercury each of radius  $R$  coalesce to form a single large drop. The ratio of total surface energy before and after the change is :

(1)  $1:2^{\frac{1}{3}}$                       (2)  $2^{\frac{1}{3}}:1$                       (3)  $2:1$                       (4)  $1:2$

**Ans.** (2)

**Sol.**  $2 \times \frac{4}{3} \pi R^3 = \frac{4}{3} \pi r^3$

$\frac{R}{r} = \left(\frac{1}{2}\right)^{1/3} \dots(1)$

Now  $\frac{U_{\text{before}}}{U_{\text{after}}} = \frac{2 \times S \times 4\pi R^2}{S \times 4\pi r^2} = 2 \times \left(\frac{R}{r}\right)^2 = \frac{2^{1/3}}{1}$

16. A body rolls down an inclined plane without slipping. The kinetic energy of rotation is 50% of its translational kinetic energy. The body is :

- (1) Hollow cylinder      (2) Solid sphere      (3) Solid cylinder      (4) Ring

Ans. (3)

Sol. Given  $\frac{1}{2}I\omega^2 = \frac{1}{2} \times \frac{1}{2}mV^2$

as  $v = R\omega$  (pure rolling)

$$\frac{1}{2}I\omega^2 = \frac{1}{4}mR^2\omega^2 ; I = \frac{1}{2}mR^2$$

Thus, solid cylinder.

17. For a series LCR circuit with  $R = 100 \Omega$ ,  $L = 0.5 \text{ mH}$  and  $C = 0.1 \text{ pF}$  connected across  $220 \text{ V} - 50 \text{ Hz AC}$  supply, the phase angle between current and supplied voltage and the nature of the circuit is :

- (1)  $0^\circ$ , resonance circuit      (2)  $\approx 90^\circ$ , predominantly capacitive circuit  
 (3)  $0^\circ$ , resistive circuit      (4)  $\approx 90^\circ$ , predominantly inductive circuit

Ans. (2)

Sol.  $\omega = 2\pi f = 100\pi$

$$R = 100\Omega, X_L = \omega L = \frac{100\pi \times 0.5}{1000} = \frac{\pi}{200}, X_C = \frac{1}{\omega C} \Rightarrow \frac{10^{12}}{100\pi \times 0.1} = \frac{10^{11}}{\pi}\Omega$$

As  $X_C \gg X_L$  &  $X_C \gg R$

Then  $\tan \phi = \frac{|X_L - X_C|}{R} \approx \infty ; \phi = 90^\circ$

18. If time ( $t$ ), velocity ( $v$ ), and angular momentum ( $\ell$ ) are taken as the fundamental units. Then the dimension of mass ( $m$ ) in terms of  $t, v$ , and  $\ell$  is :

- (1)  $[t^{-1} v^{-2} \ell^1]$       (2)  $[t^{-2} v^{-1} \ell^1]$       (3)  $[t^{-1} v^2 \ell^{-1}]$       (4)  $[t^{-1} v^1 \ell^{-2}]$

Ans. (1)

Sol.  $M \propto t^x v^y \ell^z$

$$M^0 L^0 T^0 = t^x [LT^{-1}]^y [ML^2 T^{-1}]^z$$

$$M^1 L^0 T^0 = t^{x-y-z} L^{y+2z} M^z$$

On comparing powers

$$z = 1 \quad \dots(1)$$

$$x - y - z = 0 \quad \dots(2)$$

$$y + 2z = 0 \quad \dots(3)$$

$$y + 2 \times 1 = 0$$

$$y = -2$$

$$x - (-2) - 1 = 0$$

$$x = -1 ; M \propto t^{-1} v^{-2} \ell^1 ; [M] \propto [t^{-1} v^{-2} \ell]$$

19. A particle is making simple harmonic motion along, the X-axis. If at a distances  $x_1$  and  $x_2$  from the mean position the velocities of the particle are  $v_1$  and  $v_2$  respectively. The time period of its oscillation is given as :

$$(1) T = 2\pi \sqrt{\frac{x_2^2 + x_1^2}{v_1^2 - v_2^2}} \quad (2) T = 2\pi \sqrt{\frac{x_2^2 + x_1^2}{v_1^2 + v_2^2}} \quad (3) T = 2\pi \sqrt{\frac{x_2^2 - x_1^2}{v_1^2 + v_2^2}} \quad (4) T = 2\pi \sqrt{\frac{x_2^2 - x_1^2}{v_1^2 - v_2^2}}$$

Ans. (4)

Sol.  $v = \omega \sqrt{A^2 - x^2}$

$$v_1 = \omega \sqrt{A^2 - x_1^2}$$

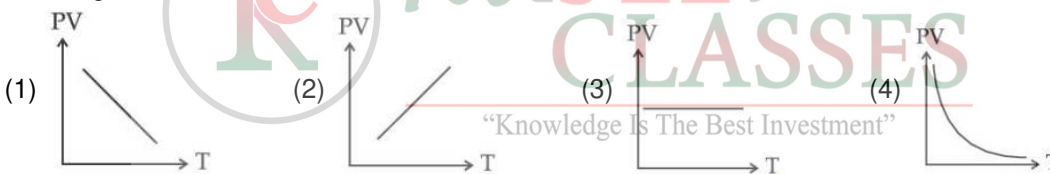
$$v_2 = \omega \sqrt{A^2 - x_2^2}$$

$$\left(\frac{v_1}{\omega}\right)^2 - \left(\frac{v_2}{\omega}\right)^2 = x_2^2 - x_1^2$$

$$\omega^2 = \frac{v_1^2 - v_2^2}{x_2^2 - x_1^2}$$

$$\omega = \sqrt{\frac{v_1^2 - v_2^2}{x_2^2 - x_1^2}} ; T = 2\pi \sqrt{\frac{x_2^2 - x_1^2}{v_1^2 - v_2^2}}$$

20. Which of the following graphs represent the behaviour of an ideal gas ? Symbols have their usual meaning.



Ans. (2)

Sol.  $PV = nRT$

$$\Rightarrow PV = CT$$

Therefore, PV v/s T graph is straight line.

### Numerical Value Type

This section contains **10 Numerical value type questions.**

1. Two bodies, a ring and a solid cylinder of same material are rolling down without slipping and inclined plane. The radii of the bodies are same. The ratio of velocity of the centre of mass at the bottom of the inclined plane of the ring to that of the cylinder is  $\frac{\sqrt{x}}{2}$ . Then, the value of x is \_\_\_\_\_.

Ans. 3

**Sol.** If we consider both the bodies have same mass

$$K.E_{ring} = K.E_{Solid\ cylinder}$$

$$\frac{1}{2}M_R V_R^2 + \frac{1}{2}I_R \omega_R^2 = \frac{1}{2}M_C V_C^2 + \frac{1}{2}I_C \omega_C^2 \quad [I_R = M_R R^2, I_C = 1/2 M_C R^2]$$

$$2M_R V_R^2 = \frac{3}{2}M_C V_C^2 \quad [\text{considering } M_R = M_C]$$

$$\frac{V_R}{V_C} = \frac{\sqrt{3}}{2}$$

2. A body of mass 'm' is launched up on a rough inclined plane making an angle of  $30^\circ$  with the horizontal. The coefficient of friction between the body and plane is  $\frac{\sqrt{x}}{5}$  if the time of ascent is half of the time of descent. The value of x is \_\_\_\_\_ .

**Ans.** 3

**Sol.**  $S = \frac{1}{2} a_A t_A^2$  .....(1)

$$S = \frac{1}{2} a_D t_D^2 \quad \text{.....(2)}$$

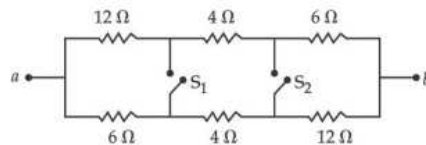
From Equation (1) & (2)

$$\frac{t_A^2}{t_D^2} = \frac{a_D}{a_A}$$

$$\Rightarrow \frac{t_A^2}{t_D^2} = \frac{g \sin \theta - \mu g \cos \theta}{g \sin \theta + \mu g \cos \theta} \Rightarrow \frac{t_A}{t_D} = \sqrt{\frac{g \sin \theta - \mu g \cos \theta}{g \sin \theta + \mu g \cos \theta}}$$

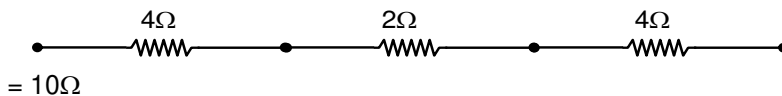
$$\Rightarrow \frac{1}{2} = \sqrt{\frac{1 - \sqrt{3}\mu}{1 + \sqrt{3}\mu}} \Rightarrow 1 + \sqrt{3}\mu = 4 - 4\sqrt{3}\mu \Rightarrow 5\sqrt{3}\mu = 3 \Rightarrow \mu = \frac{\sqrt{3}}{5}$$

3. In the given figure switches  $S_1$  and  $S_2$  are in open condition. The resistance across ab when the switches  $S_1$  and  $S_2$  are closed is \_\_\_\_\_  $\Omega$ .



**Ans.** 10

**Sol.**





4. A series LCR circuit of  $R = 5 \Omega$ ,  $L = 20 \text{ mH}$  and  $C = 0.5 \mu\text{F}$  is connected across an AC supply of 250 V, having variable frequency. The power dissipated at resonance condition is \_\_\_\_\_  $\times 10^2 \text{ W}$ .

**Ans.** 125

**Sol.** As circuit is in resonance. Thus

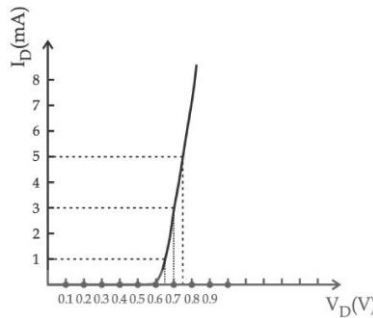
$$X_L = X_C$$

$$\therefore Z = R \text{ so } i_{\text{rms}} = V/Z = V/R$$

$$P = i_{\text{rms}}^2 R$$

$$P = \frac{V^2}{R} = \frac{250 \times 250}{5} = 12500 \text{ J/s} = 125 \times 10^2 \text{ W}$$

5. For the forward biased diode characteristics shown in the figure, the dynamic resistance at  $I_D = 3 \text{ mA}$  will be \_\_\_\_\_  $\Omega$ .



**Ans.** 25

**Sol.** Dynamic resistance =  $\frac{\Delta V}{\Delta I}$   
 $\frac{0.75 - 0.65}{(5 - 1) \times 10^{-3}} = 25$

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6. A certain metallic surface is illuminated by monochromatic radiation of wavelength  $\lambda$ . The stopping potential for photoelectric current for this radiation is  $3V_0$ . If the same surface is illuminated with a radiation of wavelength  $2\lambda$ , the stopping potential is  $V_0$ , The threshold wavelength of this surface for photoelectric effect is \_\_\_\_\_  $\lambda$ .

**Ans.** 4

**Sol.**  $KE = h\nu - W$

$$eV = \frac{hc}{\lambda} - W$$

$$\text{For first case ; } e(3V_0) = \frac{hc}{\lambda} - W \quad \dots(i)$$

$$\text{For second case; } eV_0 = \frac{hc}{2\lambda} - W \quad \dots(ii)$$

From equation (i) and (ii)

$$W = \frac{hc}{4\lambda}$$

For  $\lambda_{th}$ ;  $W = \frac{hc}{\lambda_{th}}$

$$\Rightarrow \frac{hc}{4\lambda} = \frac{hc}{\lambda_{th}} \Rightarrow \lambda_{th} = 4\lambda$$

7. A body rotating with an angular speed of 600 rpm is uniformly accelerated to 1800 rpm in 10 sec. The number of rotations made in the process is \_\_\_\_\_ .

**Ans. 200**

**Sol.**  $\omega_0 = 600 \text{ rpm} = 10 \text{ rps}$

$$\omega = 1800 \text{ rpm} = 30 \text{ rps}$$

$$\alpha = \frac{30 - 10}{10} = 2 \text{ rps}^2$$

$$\theta = \omega_0 t + \frac{1}{2} \alpha t^2 = 10 \times 10 + \frac{1}{2} \times 2 \times (100) = 200$$

8. A radioactive substance decays to  $\left(\frac{1}{16}\right)^{th}$  of its initial activity in 80 days. The half life of the radioactive substance expressed in days is \_\_\_\_\_ .

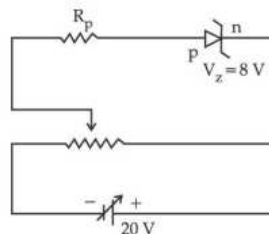
**Ans. 20**

**Sol.**  $N_0 \xrightarrow{t_{1/2}} \frac{N_0}{2} \xrightarrow{t_{1/2}} \frac{N_0}{4} \xrightarrow{t_{1/2}} \frac{N_0}{8} \xrightarrow{t_{1/2}} \frac{N_0}{16}$

$$4 \times t_{1/2} = 80 \text{ days}$$

$$t_{1/2} = 20 \text{ days}$$

9. A zener diode having zener voltage 8 v and power dissipation rating of 0.5 W is connected across a potential divider arranged with maximum potential drop across zener diode is as shown in the diagram. The value of protective resistance  $R_P$  is \_\_\_\_\_  $\Omega$ .



**Ans. 192**

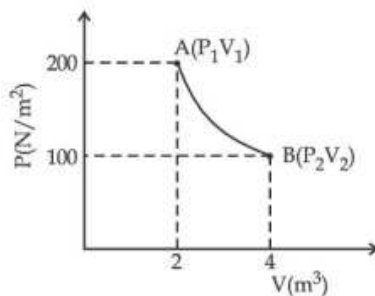
**Sol.**  $i = \frac{20}{128 + R_p}$

$$12 = \frac{20}{128 + R_p} \times R_p$$

$$1536 = 8R_p$$

$$R_p = 192$$

- 10.** One mole of an ideal gas at 27°C is taken from A to B as shown in the given PV indicator diagram. The work done by the system will be \_\_\_\_\_  $\times 10^{-1}$  J. [Given :  $R = 8.3$  J/mole K,  $\ln 2 = 0.6931$ ] (Round off to the nearest integer)

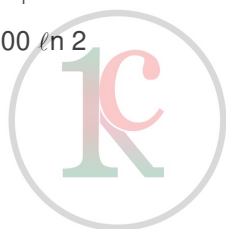


**Ans.** 17258

**Sol.**  $W = nRT \ln \frac{V_2}{V_1}$

$$= 1 \times 8.3 \times 300 \ln 2$$

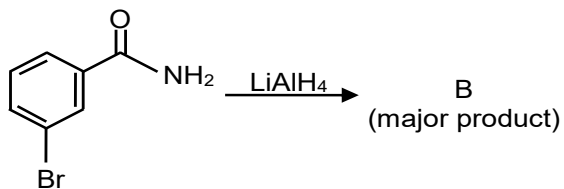
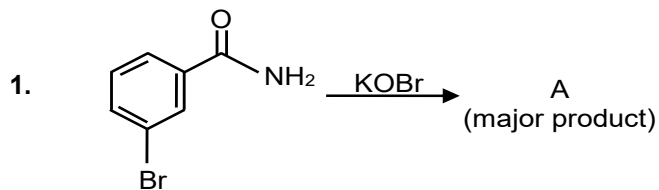
$$= 1725.8 \text{ J}$$



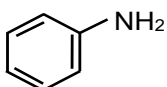
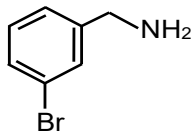
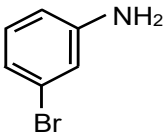
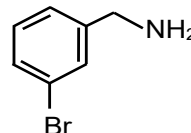
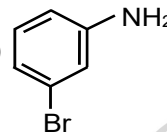
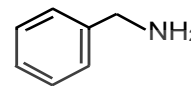
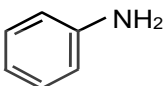
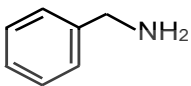
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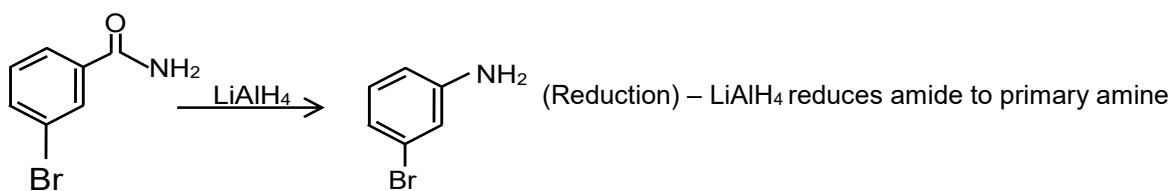
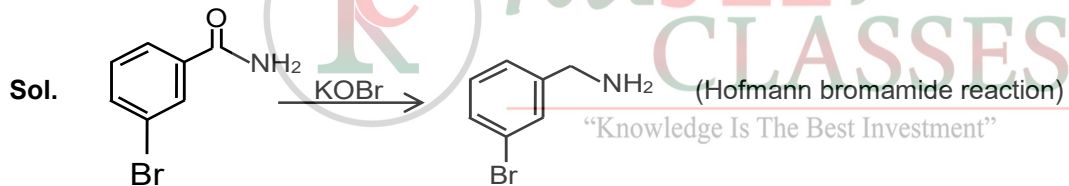
# CHEMISTRY



In the above reactions, product A and product B respectively are :

- (1)  ,  (2)  , 
- (3)  ,  (4)  , 

Ans. (2)

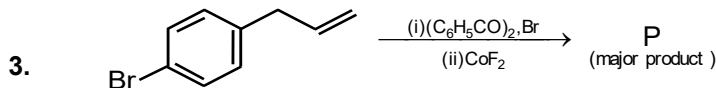


2. Metallic sodium does not react normally with :

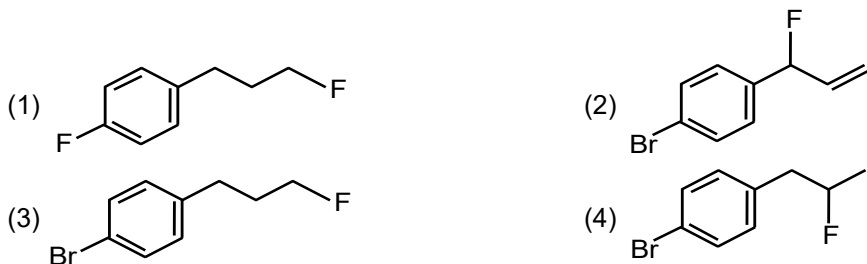
- (1) Tert-butyl alcohol (2) Gaseous ammonia  
(3) But-2-yne (4) Ethyne

Ans. (3)

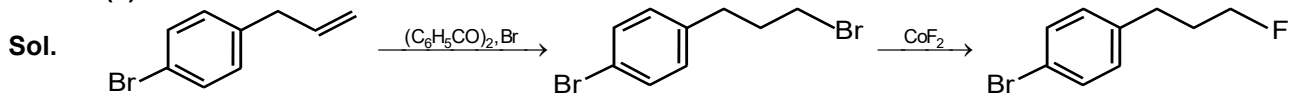
Sol. But-2-yne does not have any acidic hydrogen.



Major product P of above reaction, is :



Ans. (3)



4. Outermost electronic configuration of a group 13 element, E, is  $4s^2, 4p^1$ . The electronic configuration of an element of p-block period-five placed diagonally to element, E is :



Ans. (4)

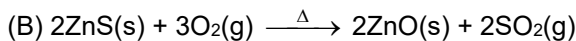
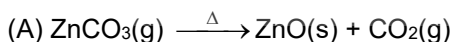
Sol.

|                        | 13 <sup>th</sup> | 14 <sup>th</sup> | 15 <sup>th</sup> |
|------------------------|------------------|------------------|------------------|
| 2 <sup>nd</sup> period | $2s^2 2p^1$ B    | C                | N                |
| 3 <sup>rd</sup> period | $3s^2 3p^1$ Al   | Si               | P                |
| 4 <sup>th</sup> period | $4s^2 4p^1$ Ga   | Ge               | As               |
| 5 <sup>th</sup> period | $5s^2 5p^1$ In   | Sn               | Sb               |

That element is  ${}_{50}\text{Sn} \Rightarrow [\text{Kr}] 4d^{10} 5s^2 5p^2$

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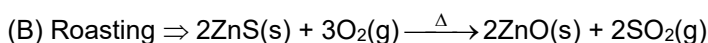
5. Consider two chemical reactions (A) and (B) that take place during metallurgical process :



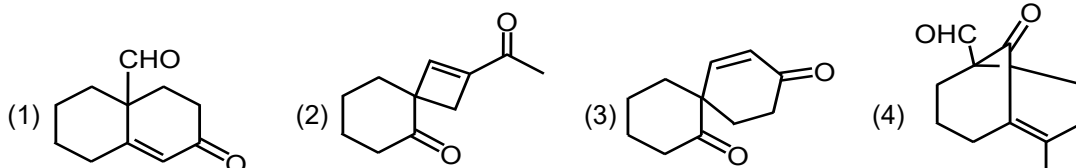
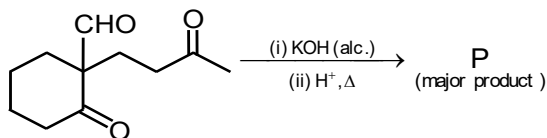
The correct option of names give to them respectively is :

- (1) Both (A) and (B) are producing same product so both are calcination  
 (2) Both (A) and (B) are producing same product so both are roasting  
 (3) (A) is roasting and (B) is calcination  
 (4) (A) is calcination and (B) is roasting

Ans. (4)



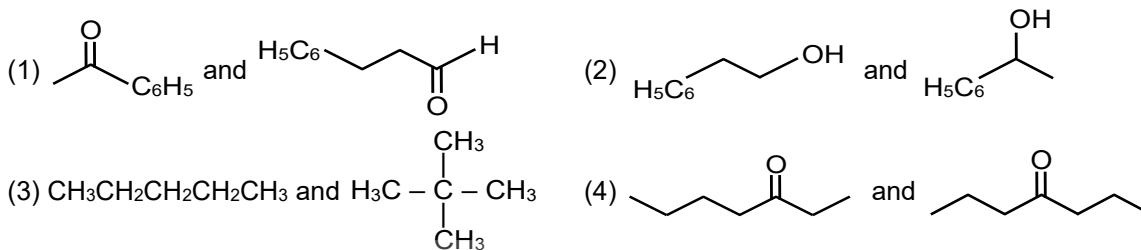
6. The major product (P) in the following reaction is :



Ans. (3)

Sol. Intramolecular aldol condensation.

7. Which one of the following pairs of isomers is an example of metamerism ?



Ans. (4)

Sol. Metamers are compounds which have different alkyl groups present along both side of polyvalent functional group.



8. The single largest industrial application of dihydrogen is :

- (1) Rocket fuel in space research (2) Manufacture of metal hydrides  
 (3) In the synthesis of ammonia (4) In the synthesis of nitric acid

Ans. (3)

Sol. The largest single industrial application of dihydrogen in the synthesis of ammonia [NCERT page 287]

9. Which one of the following statement is **not** true about enzymes ?

- (1) Enzymes are non-specific for a reaction and substrate.  
 (2) Almost all enzymes are proteins  
 (3) The action of enzymes is temperature and pH specific  
 (4) Enzymes work as catalysis by lowering the activation energy of a biochemical reaction.

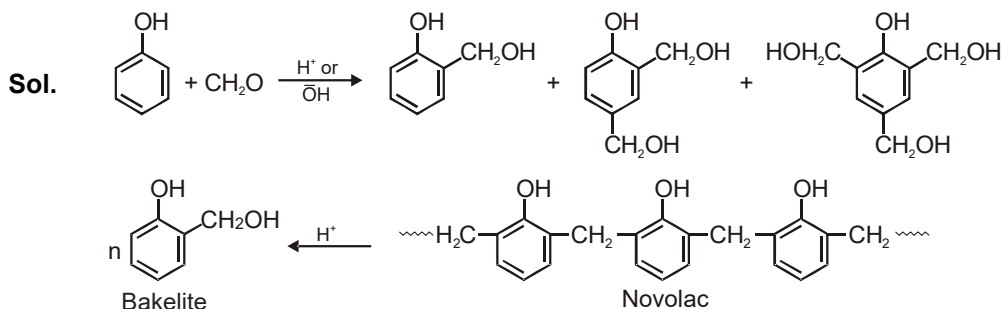
Ans. (1)

Sol. Enzymes are highly specific in nature

10. Bakelite is a cross-linked polymer of formaldehyde and :

- (1) PHBV                      (2) Dacron                      (3) Buna-S                      (4) Novolac

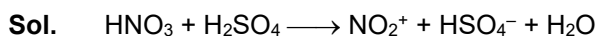
Ans. (4)



11. Benzene on nitration gives nitrobenzene in presence of HNO<sub>3</sub> and H<sub>2</sub>SO<sub>4</sub> mixture, where :

- (1) both H<sub>2</sub>SO<sub>4</sub> and HNO<sub>3</sub> act as a bases                      (2) HNO<sub>3</sub> acts as a base and H<sub>2</sub>SO<sub>4</sub> acts as an acid  
 (3) both H<sub>2</sub>SO<sub>4</sub> and HNO<sub>3</sub> act as a acids                      (4) HNO<sub>3</sub> acts as a acid and H<sub>2</sub>SO<sub>4</sub> acts as a base

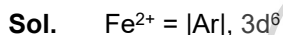
Ans. (2)



12. Spin only magnetic moment of an octahedral complex of Fe<sup>2+</sup> in the presence of a strong filed ligand in BM is :

- (1) 0                                      (2) 2.82                                      (3) 4.89                                      (4) 3.46

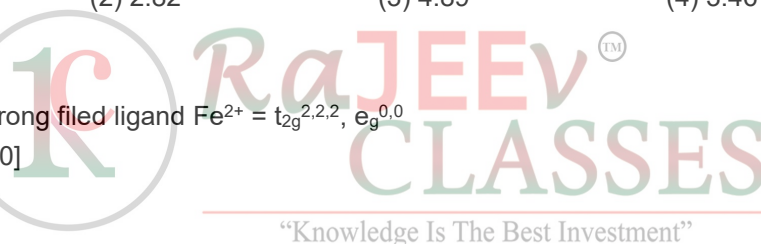
Ans. (1)



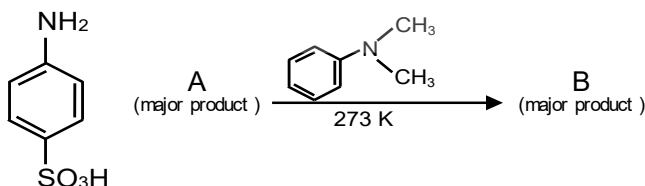
in presence of strong filed ligand  $\text{Fe}^{2+} = t_{2g}^{2,2,2}, e_g^{0,0}$

Unpaired e<sup>-</sup> [n = 0]

$\mu = \sqrt{n(n+2)} = 0$



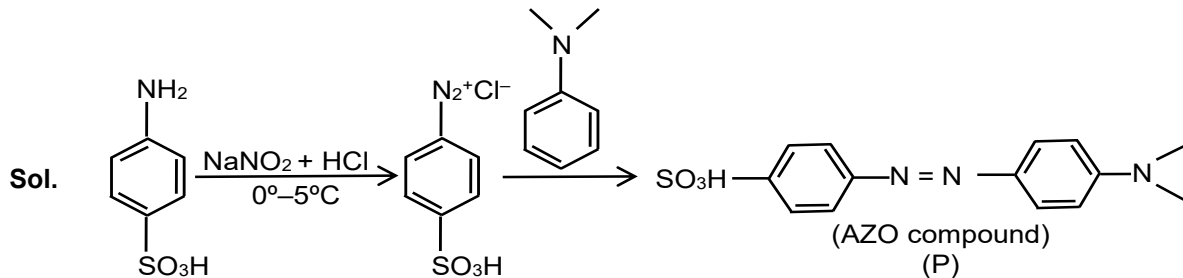
13.



Consider the above reaction, compound B is :

- (1)
- (2)
- (3)
- (4)

Ans. (2)



Diazotisation reaction

14. Which one of the following gases is reported to retard photosynthesis ?

- (1) CO<sub>2</sub>                      (2) CO                      (3) CFCs                      (4) NO<sub>2</sub>

Ans. (4)

Sol. Reason: NO<sub>2</sub> damage the leaves of plants and retard the photosynthesis.

15. The hybridisation of the atomic orbitals of nitrogen in NO<sub>2</sub><sup>-</sup>, NO<sub>2</sub><sup>+</sup> and NH<sub>4</sub><sup>+</sup> respectively are :

- (1) sp<sup>2</sup>, sp and sp<sup>3</sup>              (2) sp, sp<sup>2</sup> and sp<sup>3</sup>              (3) sp<sup>3</sup>, sp and sp<sup>2</sup>              (4) sp<sup>3</sup>, sp<sup>2</sup> and sp

Ans. (1)



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16. In Carius method, halogen containing organic compound is heated with fuming nitric acid in the presence of :

- (1) BaSO<sub>4</sub>                      (2) HNO<sub>3</sub>                      (3) AgNO<sub>3</sub>                      (4) CuSO<sub>4</sub>

Ans. (3)

17. Cu<sup>2+</sup> salt reacts with potassium iodide to give :

- (1) CuI                      (2) Cu<sub>2</sub>I<sub>2</sub>                      (3) Cu(I<sub>3</sub>)<sub>2</sub>                      (4) Cu<sub>2</sub>I<sub>3</sub>

Ans. (2)

Sol. 2Cu<sup>2+</sup> + 4KI → Cu<sub>2</sub>I<sub>2</sub>(S) + I<sub>2</sub> + 4K<sup>+</sup>



18. A solution is 0.1 M in  $\text{Cl}^-$  and 0.001 M in  $\text{CrO}_4^{2-}$ . Solid  $\text{AgNO}_3$  is gradually added to it. Assuming that the addition does not change in volume and  $K_{\text{sp}}(\text{AgCl}) = 1.7 \times 10^{-10} \text{ M}^2$  and  $K_{\text{sp}}(\text{Ag}_2\text{CrO}_4) = 1.9 \times 10^{-12} \text{ M}^3$ . Select correct statement from the following :
- (1)  $\text{AgCl}$  will precipitate first as the amount of  $\text{Ag}^+$  needed to precipitate is low.
  - (2)  $\text{AgCl}$  precipitate first because its  $K_{\text{sp}}$  is high.
  - (3)  $\text{Ag}_2\text{CrO}_4$  precipitates first because the amount of  $\text{Ag}^+$  needed is low.
  - (4)  $\text{Ag}_2\text{CrO}_4$  precipitate first as its  $K_{\text{sp}}$  is low.

Ans. (1)

Sol. (i) Concentration of  $\text{Ag}^+$  required for precipitation of  $\text{AgCl}$

$$K_{\text{sp}}(\text{AgCl}) = [\text{Ag}^+][\text{Cl}^-]$$

$$1.7 \times 10^{-10} = [\text{Ag}^+](0.1)$$

$$[\text{Ag}^+] = 1.7 \times 10^{-9} \text{ M}$$

(ii) Concentration of  $\text{Ag}^+$  required for precipitation of  $\text{Ag}_2\text{CrO}_4$

$$K_{\text{sp}}(\text{Ag}_2\text{CrO}_4) = [\text{Ag}^+]^2 [\text{CrO}_4^{2-}]$$

$$1.9 \times 10^{-12} = [\text{Ag}^+]^2 (0.001)$$

$$[\text{Ag}^+] = \sqrt{1.9 \times 10^{-9}} = \sqrt{19} \times 10^{-5}$$

So,  $\text{AgCl}$  get precipitated first.

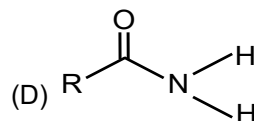
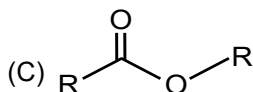
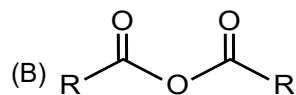
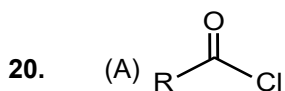
19. Which one of the following species **doesn't** have a magnetic moment of 1.73 BM, (spin only value) ?

- (1)  $\text{O}_2^-$                       (2)  $\text{CuI}$                       (3)  $\text{O}_2^+$                       (4)  $[\text{Cu}(\text{NH}_3)_4]\text{Cl}_2$

Ans. (2)

Sol.  $\mu = 1.73 \text{ BM}$  It means number of unpaired electron = 1

| Species          | Unpaired electron |
|------------------|-------------------|
| $\text{O}_2^-$   | 1                 |
| $\text{O}_2^+$   | 1                 |
| $\text{Cu}^+$    | 0                 |
| $\text{Cu}^{2+}$ | 1                 |



The correct order of their reactivity towards hydrolysis at room temperature is :

- (1) (D) > (A) > (B) > (C)                      (2) (A) > (C) > (B) > (D)  
 (3) (D) > (B) > (A) > (C)                      (4) (A) > (B) > (C) > (D)

Ans. (4)

Sol. Rate of hydrolysis is directly proportional to  $\delta$  positive charged present on carbon of  $\text{C}=\text{O}$  group.

Rate of hydrolysis – Acid chloride > Acid anhydride > ester > amide

## Section : Chemistry Section B

1. The vapour pressures of A and B at 25°C are 90 mm Hg and 15 mm Hg respectively. If A and B are mixed such that the mole fraction of A in the mixture is 0.6, then the mole fraction of B in the vapour phase is  $x \times 10^{-1}$ . The value of x is \_\_\_\_\_. (Nearest integer)

**Ans. (1)**

**Sol.**  $P_A^0 = 90$  &  $P_B^0 = 15$

$$P_{\text{total}} = 90 \times 0.6 + (15) \times 0.4$$

$$= 54 + 6 = 60 \text{ mm of Hg}$$

$$P_B = P_{\text{total}} Y_B = P_B^0 Y_B$$

$$Y_B = \frac{15 \times 0.4}{60} = 0.1$$

$$= 1 \times 10^{-1}$$

**Ans. 1**

2. 4 g equimolar mixture of NaOH and  $\text{Na}_2\text{CO}_3$  contains x g of NaOH and y g of  $\text{Na}_2\text{CO}_3$ . The value of x is \_\_\_\_\_ g. (Nearest integer)

**Ans. (1)**

**Sol.** Given (i)  $x + y = 4$

$$(ii) \frac{X}{40} = \frac{Y}{106} \text{ [Equimolar]}$$

$$Y = \left[ \frac{106}{40} \right] X$$

So  $X + \frac{106}{40} X = 4$

$$X + 2.65X = 4$$

$$3.65X = 4$$

$$X = 1.096 \text{ gram.}$$

3. For a given chemical reaction  $A \rightleftharpoons B$  at 300 K the free energy change is  $-49.4 \text{ kJ mol}^{-1}$  and the enthalpy of reaction is  $51.4 \text{ kJ mol}^{-1}$ . The entropy change of the reaction is \_\_\_\_\_  $\text{JK}^{-1} \text{ mol}^{-1}$ . (Nearest integer)

**Ans. (336)**

**Sol.**  $\Delta G^\circ = \Delta H^\circ - T \Delta S^\circ$

$$-49.4 = 51.4 - T \Delta S^\circ$$

$$-\Delta S^\circ = -\left[ \frac{49.4 + 51.4}{300} \right]$$

$$= 0.336 \text{ KJ/K} = 336 \text{ J/K}$$

4. Diamond has a three dimensional structure of C atoms formed by covalent bonds. The structure of diamond has face centred cubic lattice where 50% of the tetrahedral voids are also occupied by carbon atoms. The number of carbon atoms present per unit cell of diamond is \_\_\_\_\_.

**Ans. (8)**

**Sol.** In diamond carbon formed : FCC unit cell + 50% (TV) are occupied

$$\text{effective number of atoms of carbon} = 4 + 8 \times \frac{1}{2} = 8$$

5. The wavelengths of electrons accelerated from rest through a potential difference of 40 kV is  $x \times 10^{-12}$  m. The value of x is \_\_\_\_\_. (Nearest integer)

Given : Mass of electron =  $9.1 \times 10^{-31}$  kg

Charge on an electron =  $1.6 \times 10^{-19}$  kg

Planck's constant =  $6.63 \times 10^{-34}$  Js

**Ans. (6)**

**Sol.**  $\lambda = \frac{12.3}{\sqrt{v}}$

$$v = 40 \times 10^3$$

$$\lambda = \frac{12.3}{\sqrt{40 \times 10^3}} = \frac{12.3}{2 \times 10^2}$$

$$= 0.0615 \text{ \AA} = 6.15 \times 10^{-12} \text{ m}$$

**Ans. 6**

6. 100 ml 0.0018% (w/v) solution of  $\text{Cl}^-$  ion was the minimum concentration of  $\text{Cl}^-$  required to precipitate a negative sol in one h. The coagulating value of  $\text{Cl}^-$  ion is \_\_\_\_\_. (Nearest integer)

**Ans. (1)**

**Sol. Coagulation value :** The minimum concentration of electrolyte in millimoles required to cause coagulation of 1 lit. of colloidal solution

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Given : 0.0018 gram  $\text{Cl}^-$  present in 100 ml solution.

$$\text{Coagulation value of } \text{Cl}^- = \frac{\left\{ \frac{0.0018}{35.5} \times 10^3 \right\}}{0.1} = 0.5070$$

**Ans. (1)**

7.  $\text{PCl}_5(\text{g}) \longrightarrow \text{PCl}_3(\text{g}) + \text{Cl}_2(\text{g})$

In the above first order reaction the concentration of  $\text{PCl}_5$  reduces from initial concentration  $50 \text{ mol L}^{-1}$  to  $10 \text{ mol L}^{-1}$  in 120 minutes at 300 K. The rate constant for the reaction at 300 K is  $x \times 10^{-2} \text{ min}^{-1}$ . The value of x is \_\_\_\_\_.

[Given  $\log 5 = 0.6989$ ]

**Ans. (1)**

**Sol.**  $\text{PCl}_5(\text{g}) \longrightarrow \text{PCl}_3(\text{g}) + \text{Cl}_2(\text{g})$

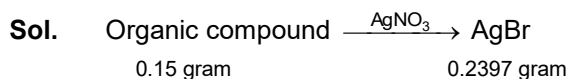
t = 0                      50 moles

t = 120 minutes 10 mole

$$\begin{aligned}
 K &= \frac{1}{t} \ln \left( \frac{a}{a-x} \right) \\
 &= \frac{2.303}{120} \log \left( \frac{50}{10} \right) \\
 &= \frac{2.303 \times 0.693}{120} = 0.0133 \text{ minutes}^{-1}. \\
 &= 1.33 \times 10^{-2} \text{ minutes}^{-1} \approx 1 \text{ minutes}^{-1}.
 \end{aligned}$$

8. When 0.15 g of an organic compound was analysed using Carius method estimation of bromine, 0.2397 g of AgBr was obtained. The percentage of bromine in the organic compound is \_\_\_\_\_. (Nearest integer) (Atomic mass : Silver = 108, Bromine = 80)

**Ans. (68)**



$$\text{Mole of AgBr} = \left\{ \frac{0.2397}{188} \right\}$$

$$\text{Mole of Br}^- = \left\{ \frac{0.2397}{188} \right\} \times 80 = 0.102 \text{ gram}$$

$$\% \text{ of Bromine in organic} = \left\{ \frac{0.102}{0.15} \right\} \times 100 = 68$$

**Ans. 68**

9. Potassium chlorate is prepared by electrolysis of KCl in basic solution as shown by following equation.



A current of xA has to be passed for 10 h to produce 10.0 g of potassium chlorate. The value of x is \_\_\_\_\_. (Nearest integer)

(Molar mass of  $\text{KClO}_3 = 122.6 \text{ g mol}^{-1}$ ,  $F = 96500 \text{ C}$ )

**Ans. (1)**



$$w = \left[ \frac{E}{96500} \right] i \times t$$

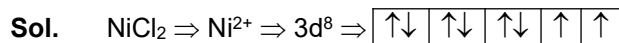
$$10 = \left[ \frac{122.6}{6 \times 96500} \right] x \times 10 \times 60 \times 60$$

$$x = 1.31 \text{ A}$$

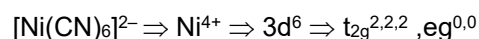
**Ans. 1**

10. An aqueous solution of  $\text{NiCl}_2$  was heated with excess sodium cyanide in presence of strong oxidizing agent to form  $[\text{Ni}(\text{CN})_6]^{2-}$ . The total change in number of unpaired electron on metal centre is \_\_\_\_\_. .

**Ans. (2)**



Unpaired electron  $n = 2$



unpaired electron = 0

# MATHEMATICS

1. Let  $y = y(x)$  satisfies the equation  $\frac{dy}{dx} - |A| = 0$ , for all  $x > 0$ , where  $A = \begin{bmatrix} y & \sin x & 1 \\ 0 & -1 & 1 \\ 2 & 0 & \frac{1}{x} \end{bmatrix}$ .

If  $y(\pi) = \pi + 2$ , then the value of  $y\left(\frac{\pi}{2}\right)$  is :

माना  $y = y(x)$  समीकरण  $\frac{dy}{dx} - |A| = 0, \forall x > 0$ , को संतुष्ट करता है, जबकि  $A = \begin{bmatrix} y & \sin x & 1 \\ 0 & -1 & 1 \\ 2 & 0 & \frac{1}{x} \end{bmatrix}$ .

यदि  $y(\pi) = \pi + 2$  है, तो  $y\left(\frac{\pi}{2}\right)$  बराबर है :

(1)  $\frac{\pi}{2} - \frac{4}{\pi}$

(2)  $\frac{3\pi}{2} - \frac{1}{\pi}$

(3)  $\frac{\pi}{2} - \frac{1}{\pi}$

(4)  $\frac{\pi}{2} + \frac{4}{\pi}$

**Ans.** (4)

**Sol.**  $|A| = y\left(-\frac{1}{x}\right) - \sin x(-2) + 1(+2)$

$|A| = -\frac{y}{x} + 2\sin x + 2$

$\Rightarrow \frac{dy}{dx} - \left(-\frac{y}{x} + 2\sin x + 2\right) = 0$

$\Rightarrow \frac{dy}{dx} + \frac{y}{x} - 2(\sin x + 1) = 0$

I.F. =  $e^{\int \frac{1}{x} dx} = x$

$yx = \int 2(\sin x + 1)xdx$

$\Rightarrow yx = 2\left[\int xdx + \int x \sin x dx\right]$

$\Rightarrow yx = 2\left[\frac{x^2}{2} + (-x \cos x + \sin x)\right] + c$

$\Rightarrow yx = x^2 - 2x \cos x + 2 \sin x + c$

At  $x = \pi, y = \pi + 2$

$\Rightarrow (\pi + 2)\pi = \pi^2 + 2\pi + c$

$\Rightarrow \pi^2 + 2\pi = \pi^2 + 2\pi + c$

$\Rightarrow c = 0$



$$\text{At } x = \frac{\pi}{2}$$

$$\Rightarrow y \cdot \frac{\pi}{2} = \left(\frac{\pi}{2}\right)^2 - 0 + 2 + 0 \Rightarrow y \cdot \frac{\pi}{2} = \left(\frac{\pi}{2}\right)^2 + 2$$

$$\Rightarrow y = \frac{\pi}{2} + \frac{4}{\pi}$$

2. If  $[x]$  denotes the greatest integer less than or equal to  $x$ , then the value of the integral  $\int_{-\pi/2}^{\pi/2} ([x] - \sin x) dx$  is equal to :

यदि  $[x]$ , महत्तम पूर्णांक  $\leq x$  है, तो समाकलन  $\int_{-\pi/2}^{\pi/2} ([x] - \sin x) dx$  का मान बराबर है :

- (1)  $-\pi$                       (2) 1                      (3)  $\pi$                       (4) 0

**Ans. (1)**

**Sol.**  $\int_{-\pi/2}^{\pi/2} ([x] - \sin x) dx$

$$= \int_{-\pi/2}^{\pi/2} ([x] - \sin x) dx$$

$$= \int_{-\pi/2}^{\pi/2} ([-\sin x] + [x]) dx \quad (\because [x+1] = [x]+1)$$

Use property  $\int_{-a}^a f(x) dx = \int_0^a (f(x) + f(-x)) dx$

$$= \int_0^{\pi/2} ([-\sin x] + [x] + [\sin x] + [-x]) dx$$

$$= \int_0^{\pi/2} (-1 - 1) dx \quad \{\because [x] + [-x] = -1, x \notin \mathbb{I}\}$$

$$= -2 (x)_0^{\pi/2}$$

$$= -\pi$$



3. The lines  $x = ay - 1 = z - 2$  and  $x = 3y - 2 = bz - 2$ , ( $ab \neq 0$ ) are coplaner if :

रेखाएँ  $x = ay - 1 = z - 2$  तथा  $x = 3y - 2 = bz - 2$ , ( $ab \neq 0$ ) समतलीय हैं, यदि :

- (1)  $a = 1, b \in \mathbb{R} - \{0\}$       (2)  $a = 2, b = 3$       (3)  $b = 1, a \in \mathbb{R} - \{0\}$       (4)  $a = 2, b = 2$

Ans. (3)

Sol. 
$$\frac{x-0}{1} = \frac{y-\frac{1}{a}}{\frac{1}{a}} = \frac{z-2}{1} \quad \dots\dots(1)$$

$$\frac{x-0}{1} = \frac{y-\frac{2}{3}}{\frac{1}{3}} = \frac{z-2}{\frac{1}{b}} \quad \dots\dots(2)$$

For coplanar :

$$\begin{vmatrix} a_2 - a_1 & b_2 - b_1 & c_2 - c_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$

$$\begin{vmatrix} 0 & \frac{2}{3} - \frac{1}{a} & \frac{2}{b} - 2 \\ 1 & \frac{1}{a} & 1 \\ 1 & \frac{1}{3} & \frac{1}{b} \end{vmatrix} = 0$$

$$\Rightarrow -\left(\frac{2}{3} - \frac{1}{a}\right)\left\{\frac{1}{b} - 1\right\} + \left(\frac{2}{b} - 2\right)\left\{\frac{1}{3} - \frac{1}{a}\right\} = 0$$

$$\Rightarrow \left(\frac{1}{a} - \frac{2}{3}\right)\left(\frac{1}{b} - 1\right) + \left(\frac{2}{b} - 2\right)\left(\frac{1}{3} - \frac{1}{a}\right) = 0$$

$$\Rightarrow (3 - 2a)(1 - b) + (2 - 2b)(a - 3) = 0$$

$$\Rightarrow 3 - 3b - 2a + 2ab + 2a - 6 - 2ab + 6b = 0$$

$$\Rightarrow 3b - 3 = 0$$

$$\Rightarrow b = 1, a \in \mathbb{R} - \{0\}$$

4. Let in a right angled triangle, the smallest angle be  $\theta$ . If a triangle formed by taking the reciprocal of its sides is also a right angled triangle, then  $\sin\theta$  is equal to :

माना एक समकोण त्रिभुज में, सबसे छोटा कोण  $\theta$  है। यदि इसकी भुजाओं के व्युत्क्रम भी एक एकसमकोण त्रिभुज बनाते हैं, तो  $\sin\theta$  बराबर है :

(1)  $\frac{\sqrt{5}-1}{4}$

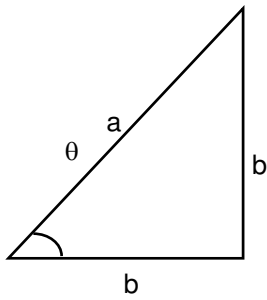
(2)  $\frac{\sqrt{5}+1}{4}$

(3)  $\frac{\sqrt{5}-1}{2}$

(4)  $\frac{\sqrt{2}-1}{2}$

Ans. (3)

Sol. Let  $a > b > c$



$$\sin\theta = \frac{c}{a}$$

$$\frac{1}{a} < \frac{1}{b} < \frac{1}{c}$$

$$\frac{1}{c^2} = \frac{1}{a^2} + \frac{1}{b^2}$$

$$1 = \frac{c^2}{a^2} + \frac{c^2}{b^2}$$

$$1 = \frac{c^2}{a^2} + \frac{c^2}{a^2 - c^2} \quad [\text{As } a^2 = b^2 + c^2]$$

$$1 = \sin^2\theta + \frac{1}{\frac{a^2}{c^2} - 1} = \sin^2\theta + \frac{1}{\operatorname{cosec}^2\theta - 1}$$

$$1 = \frac{1 - \sin^2\theta + 1}{\operatorname{cosec}^2\theta - 1} \Rightarrow \sin^2\theta + \operatorname{cosec}^2\theta = 3$$

Let  $\sin^2\theta = t$

$$t + \frac{1}{t} = 3$$

solving above equation we get

$$t = \frac{3 - \sqrt{5}}{2}, \frac{3 + \sqrt{5}}{2} \quad (\text{rejected})$$

$$\text{so, } \sin\theta = \sqrt{\frac{3 - \sqrt{5}}{2}} \Rightarrow \sin\theta = \frac{\sqrt{5} - 1}{2}$$





5. Let  $g(t) = \int_{-\pi/2}^{\pi/2} \cos\left(\frac{\pi}{4}t + f(x)\right) dx$ , where  $f(x) = \log_e\left(x + \sqrt{x^2 + 1}\right)$ ,  $x \in \mathbb{R}$ . Then which one of the following is correct ?

माना  $g(t) = \int_{-\pi/2}^{\pi/2} \cos\left(\frac{\pi}{4}t + f(x)\right) dx$ , जबकि  $f(x) = \log_e\left(x + \sqrt{x^2 + 1}\right)$ ,  $x \in \mathbb{R}$ , है। तो निम्न में से कौन सा सही है ?

- (1)  $\sqrt{2}g(1) = g(0)$       (2)  $g(1) + g(0) = 0$       (3)  $g(1) = \sqrt{2}g(0)$       (4)  $g(1) = g(0)$

**Ans. (1)**

**Sol.**  $g(t) = \int_{-\pi/2}^{\pi/2} \cos\left(\frac{\pi}{4}t + f(x)\right) dx$ ,  $f(x) = \log_e\left(x + \sqrt{x^2 + 1}\right)$

Put  $t = 1$  in  $g(t)$ , we get

$$g(1) = \int_{-\pi/2}^{\pi/2} \cos\left[\frac{\pi}{4} + \log_e\left(x + \sqrt{x^2 + 1}\right)\right] dx$$

Put  $t = 0$  in  $g(t)$ , we get

$$g(0) = \int_{-\pi/2}^{\pi/2} \cos\left[\log_e\left(x + \sqrt{x^2 + 1}\right)\right] dx$$

$$g(1) = \int_{-\pi/2}^{\pi/2} \left[ \frac{1}{\sqrt{2}} \cos\left(\log_e\left(x + \sqrt{x^2 + 1}\right)\right) - \frac{1}{\sqrt{2}} \sin\left(\log_e\left(x + \sqrt{x^2 + 1}\right)\right) \right] dx$$

since,  $\left[ \sin\left(\log_e\left(x + \sqrt{x^2 + 1}\right)\right) \right]$  is odd function

$$\text{So } g(1) = \frac{1}{\sqrt{2}} \int_{-\pi/2}^{\pi/2} \cos\left(\log_e\left(x + \sqrt{x^2 + 1}\right)\right) dx$$

$$\sqrt{2}g(1) = g(0)$$

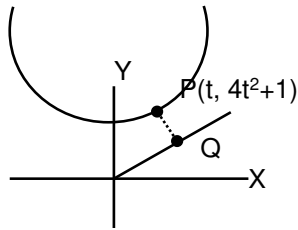
6. Let P be a variable point on the parabola  $y = 4x^2 + 1$ . Then, the locus of the mid-point of the point P and the foot of the perpendicular drawn from the point P to the line  $y = x$  is :

माना परवलय,  $y = 4x^2 + 1$  पर एक चर बिन्दु P है। तो P से रेखा,  $y = x$  पर डाले गए लम्ब के पाद तथा P के मध्य बिन्दु का बिन्दु पथ है :

- (1)  $2(3x - y)^2 + (x - 3y) + 2 = 0$       (2)  $2(x - 3y)^2 + (3x - y) + 2 = 0$   
 (3)  $(3x - y)^2 + (x - 3y) + 2 = 0$       (4)  $(3x - y)^2 + 2(x - 3y) + 2 = 0$

**Ans. (1)**

Sol.



Let  $P(t, 4t^2 + 1)$

foot of perpendiculars from P to  $y = x$  is

$$Q\left(\frac{4t^2 + t + 1}{2}, \frac{4t^2 + t + 1}{2}\right)$$

$\Rightarrow$  mid point of p and Q is

$$M\left(\frac{4t^2 + 3t + 1}{4}, \frac{12t^2 + t + 3}{4}\right)$$

Locus of M is  $2(3x - y)^2 + (x - 3y) + 2 = 0$

7. The value of  $\tan\left(2\tan^{-1}\left(\frac{3}{5}\right) + \sin^{-1}\left(\frac{5}{13}\right)\right)$  is equal to :

$\tan\left(2\tan^{-1}\left(\frac{3}{5}\right) + \sin^{-1}\left(\frac{5}{13}\right)\right)$  का मान बराबर है :

(1)  $\frac{-181}{69}$       (2)  $\frac{220}{21}$       (3)  $\frac{151}{63}$       (4)  $\frac{-291}{76}$

Ans. (2)

Sol.  $\tan\left(\tan^{-1}\frac{6}{5} + \tan^{-1}\frac{5}{12}\right)$

$$\tan\left(\tan^{-1}\left(\frac{15}{8}\right) + \tan^{-1}\left(\frac{5}{12}\right)\right) = \frac{\frac{15}{8} + \frac{5}{12}}{1 - \frac{15}{8} \cdot \frac{5}{12}} = \frac{220}{21}$$

8. If  $f : \mathbb{R} \rightarrow \mathbb{R}$  is given by  $f(x) = x + 1$ , then the value of

$$\lim_{n \rightarrow \infty} \left[ f(0) + f\left(\frac{5}{n}\right) + f\left(\frac{10}{n}\right) + \dots + f\left(\frac{5(n-1)}{n}\right) \right]$$

यदि  $f : \mathbb{R} \rightarrow \mathbb{R}$ ,  $f(x) = x + 1$  द्वारा परिभाषित है, तो

$$\lim_{n \rightarrow \infty} \left[ f(0) + f\left(\frac{5}{n}\right) + f\left(\frac{10}{n}\right) + \dots + f\left(\frac{5(n-1)}{n}\right) \right] \text{ का मान है :}$$

- (1)  $\frac{3}{2}$       (2)  $\frac{7}{2}$       (3)  $\frac{5}{2}$       (4)  $\frac{1}{2}$

**Ans. (2)**

**Sol.** 
$$= \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=0}^{n-1} f\left(\frac{5r}{n}\right) = \int_0^1 f(5x) dx = \int_0^1 (5x+1) dx$$
$$= \left(\frac{5x^2}{2} + x\right) \Big|_0^1 = \frac{5}{2} + 1 = \frac{7}{2} \text{ Ans.}$$

9. Let A, B and C be three events such that the probability that exactly one of A and B occurs is  $(1 - k)$ , the probability that exactly one of B and C occurs is  $(1 - 2k)$ , the probability that exactly one of C and A occurs is  $(1 - k)$  and the probability of all A, B and C occur simultaneously is  $k^2$ , where  $0 < k < 1$ . Then the probability that at least one of A, B and C occur is:

(1) greater than  $\frac{1}{2}$

(2) exactly equal to  $\frac{1}{2}$

(3) greater than  $\frac{1}{4}$  but less than  $\frac{1}{2}$

(4) greater than  $\frac{1}{8}$  but less than  $\frac{1}{4}$

माना A, B तथा C तीन घटनाएँ हैं जिनके लिए A और B में से ठीक एक के होने की प्रायिकता  $(1 - k)$  है, B तथा C में से ठीक एक के होने की प्रायिकता  $(1 - 2k)$  है, C तथा A में से ठीक एक के होने की प्रायिकता  $(1 - k)$  है तथा A, B और C तीनों के एक साथ होने की प्रायिकता  $k^2$  है, जबकि  $0 < k < 1$  है। तो A, B तथा C में से कम से कम एक के होने की प्रायिकता :

(1)  $\frac{1}{2}$  से बड़ी है

(2) ठीक  $\frac{1}{2}$  के बराबर है

(3)  $\frac{1}{4}$  से बड़ी परन्तु  $\frac{1}{2}$  से छोटी है

(4)  $\frac{1}{8}$  से बड़ी परन्तु  $\frac{1}{4}$  से छोटी है

**Ans. (1)**

**Sol.** 
$$P(A) + P(B) - 2P(A \cap B) = 1 - K$$
$$P(A) + P(C) - 2P(A \cap C) = 1 - 2K$$
$$P(B) + P(C) - 2P(B \cap C) = 1 - K$$
$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$
$$= \frac{3 - 4k}{2} + k^2 = \frac{2k^2 - 4k + 3}{2}$$

∴ The value of  $2k^2 - 4k + 3$  is greater than 1

∴  $P(A \cup B \cup C) > \frac{1}{2}$

10. Let  $r_1$  and  $r_2$  be the radii of the largest and smallest circles, respectively, which pass through the point  $(-4, 1)$  and having their centres on the circumference of the circle

$$x^2 + y^2 + 2x + 4y - 4 = 0. \text{ If } \frac{r_1}{r_2} = a + b\sqrt{2}, \text{ then } a + b \text{ is equal to :}$$

माना सबसे बड़े तथा सबसे छोटे वृत्तों, जो बिन्दु  $(-4, 1)$  से होकर जाते हैं तथा जिनके केन्द्र, वृत्त  $x^2 + y^2 + 2x + 4y -$

$4 = 0$  की परिधि पर स्थित हैं, की त्रिज्याएँ क्रमशः  $r_1$  तथा  $r_2$  हैं। यदि  $\frac{r_1}{r_2} = a + b\sqrt{2}$  है, तो  $a + b$  बराबर है :

- (1) 7 (2) 11 (3) 5 (4) 3

Ans. (3)

Sol.  $x^2 + y^2 + 2x + 4y - 4 = 0$

$$(x + 1)^2 + (y + 2)^2 = 3^2$$

General point on circumference  $(3 \cos \theta - 1, 3 \sin \theta - 2)$

As centre of circle is  $(-1, -2)$

$$\text{So, } r = \sqrt{(3 \cos \theta + 3)^2 + (3 \sin \theta - 3)^2}$$

$$= 3\sqrt{\cos^2 \theta + 1 + 2 \cos \theta + \sin^2 \theta + 1 - 2 \sin \theta}$$

$$= 3\sqrt{3 + 2(\cos \theta - \sin \theta)}$$

$$r_1 = 3\sqrt{3 + 2\sqrt{2}}$$

$$r_2 = 3\sqrt{3 - 2\sqrt{2}}$$

$$\frac{r_1}{r_2} = \frac{3\sqrt{3 + 2\sqrt{2}}}{3\sqrt{3 - 2\sqrt{2}}} = 3 + 2\sqrt{2}$$

On comparing with  $\frac{r_1}{r_2} = a + b\sqrt{2}$

$$a + b = 5$$

11. Let  $f : \mathbb{R} - \left\{ \frac{\alpha}{6} \right\} \rightarrow \mathbb{R}$  be defined by  $f(x) = \frac{5x + 3}{6x - \alpha}$ . Then the value of  $a$  for which  $(f \circ f)(x) = x$ , for all

$x \in \mathbb{R} - \left\{ \frac{\alpha}{6} \right\}$ , is :

- (1) No such  $\alpha$  exists (2) 6 (3) 8 (4) 5

माना  $f : \mathbb{R} - \left\{ \frac{\alpha}{6} \right\} \rightarrow \mathbb{R}$ ,  $f(x) = \frac{5x + 3}{6x - \alpha}$  द्वारा परिभाषित है। तो  $\alpha$  का मान जिसके लिए,  $(f \circ f)(x) = x$ ,  $\forall x \in \mathbb{R}$

$-\left\{ \frac{\alpha}{6} \right\}$ , है, है :

- (1) ऐसे किसी  $\alpha$  का अस्तित्व नहीं है (2) 6 (3) 8 (4) 5

Ans. (4)



**Sol.**  $f(x) = \frac{5x+3}{6x-\alpha}$

$$f(f(x)) = \frac{5\left(\frac{5x+3}{6x-\alpha}\right) + 3}{6\left(\frac{5x+3}{6x-\alpha}\right) - \alpha} = x$$

$$= \frac{25x + 15 + 18x - 3\alpha}{30x + 18 - 6x\alpha + \alpha^2} = x$$

$$\Rightarrow 25x + 15 + 18x - 3\alpha = 30x^2 + 18x - 6\alpha x^2 + \alpha^2 x$$

$$\Rightarrow 25x + 15 - 3\alpha = 30x^2 - 6\alpha x^2 + \alpha^2 x$$

$$\Rightarrow (30 - 6\alpha)x^2 + (\alpha^2 - 25)x + 3\alpha - 15 = 0$$

$$\Rightarrow 6(5 - \alpha)x^2 + (\alpha - 5)(\alpha + 5)x + 3(\alpha - 5) = 0$$

$$\Rightarrow \alpha = 5$$

**12.** The sum of all the local minimum values of the twice differentiable function  $f : \mathbb{R} \rightarrow \mathbb{R}$  defined by  $f(x) =$

$$x^3 - 3x^2 - \frac{3f''(2)}{2}x + f''(1)$$
 is :

एक फलन  $f : \mathbb{R} \rightarrow \mathbb{R}$ ,  $f(x) = x^3 - 3x^2 - \frac{3f''(2)}{2}x + f''(1)$  द्वारा परिभाषित है तथा वह दो बार अवकलनीय है। इसके

सभी स्थानीय न्यूनतम मानों का योग है :

(1) -27

(2) -22

(3) 5

(4) 0

**Ans.** (1)

**Sol.**  $f(x) = 3x^2 - 6x - \frac{3}{2}f''(2)$

$$f'(x) = 6x - 6$$

$$f'(1) = 0 \text{ \& } f'(2) = 6$$

Then the local minimum value  $f'(x) = 0$

$$\Rightarrow 3(x^2 - 2x - 3) = 0$$

$$x = -1 \text{ and } x = 3$$

$$\begin{array}{c} + \quad - \quad + \\ \hline -1 \quad 3 \end{array}$$

Local minimum at  $x = 3$

So local minimum value  $y(3) = f(3)$

$$= 3^3 - 3^3 - 3 \times \frac{6}{2} \times 3 + 0 = -27$$

13. If the mean and variance of six observations 7, 10, 11, 15, a, b are 10 and  $\frac{20}{3}$ , respectively, then the value of  $|a - b|$  is equal to :

यदि 6 प्रक्षणों, 7, 10, 11, 15, a तथा b का माध्य तथा प्रसरण क्रमशः 10 तथा  $\frac{20}{3}$  हैं, तो  $|a - b|$  का मान बराबर है :

- (1) 9 (2) 7 (3) 11 (4) 1

Ans. (4)

Sol. Mean = 10

$$\frac{7+10+11+15+a+b}{6} = 10$$

$$a + b = 17 \quad \dots\dots\dots(1)$$

$$\text{Variance} = \frac{20}{3}$$

$$\frac{49+100+121+225+a^2+b^2}{6} - 100 = \frac{20}{3}$$

$$a^2 + b^2 = 145 \quad \dots\dots\dots(2)$$

$$(a + b)^2 = 289$$

$$ab = 72$$

$$(a - b)^2 = (a + b)^2 - 4ab$$

$$(a - b)^2 = 289 - 288 = 1$$

$$|a - b| = 1$$

14. In a triangle ABC, if  $|\vec{BC}| = 3, |\vec{CA}| = 5$  and  $|\vec{BA}| = 7$ , then the projection of the vector  $\vec{BA}$  on  $\vec{BC}$  is equal to :

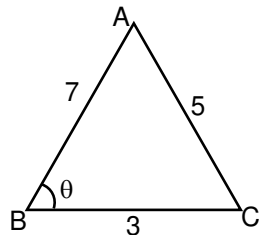
एक त्रिभुज ABC में, यदि  $|\vec{BC}| = 3, |\vec{CA}| = 5$  तथा  $|\vec{BA}| = 7$  है, तो सदिश  $\vec{BA}$  का  $\vec{BC}$  पर प्रक्षेप बराबर है :

“Knowledge Is The Best Investment”

- (1)  $\frac{11}{2}$  (2)  $\frac{13}{2}$  (3)  $\frac{15}{2}$  (4)  $\frac{19}{2}$

Ans. (1)

Sol.



Projection of  $\vec{BA}$  on  $\vec{BC} = 7 \cos\theta$

$$= 7 \left[ \frac{7^2 + 3^2 - 5^2}{2 \times 7 \times 3} \right] = \frac{11}{2}$$

15. If the real part of the complex number  $(1 - \cos\theta + i2\sin\theta)^{-1}$  is  $\frac{1}{5}$  for  $\theta \in (0, \pi)$ , then the value of the

integral  $\int_0^\theta \sin x \, dx$  is equal to :

यदि  $\theta \in (0, \pi)$  के लिए सम्मिश्र संख्या  $(1 - \cos\theta + i2\sin\theta)^{-1}$  का वास्तविक भाग  $\frac{1}{5}$  है, तो समाकलन  $\int_0^\theta \sin x \, dx$  का

मान बराबर है:

(1) 0

(2) 1

(3) -1

(4) 2

Ans. (2)

Sol.  $z = (1 - \cos\theta + 2i \sin\theta)^{-1}$

$$z = \frac{1}{1 - \cos\theta + i2\sin\theta} \times \frac{(1 - \cos\theta - i2\sin\theta)}{(1 - \cos\theta - i2\sin\theta)}$$

$$z = \frac{1 - \cos\theta - i2\sin\theta}{(1 - \cos\theta)^2 + 4\sin^2\theta}$$

$$\text{Re}(z) = \frac{1 - \cos\theta}{(1 - \cos\theta)^2 + 4\sin^2\theta} = \frac{1}{5}$$

$$\Rightarrow \theta = 0, \frac{\pi}{2}$$

But  $\theta \in (0, \pi)$

$$\text{So, } \theta = \frac{\pi}{2}$$

$$\text{Now, } \int_0^{\frac{\pi}{2}} \sin x \, dx = 1.$$



16. The value of  $k \in \mathbb{R}$ , for which the following system of linear equations

$k \in \mathbb{R}$  का वह मान, जिसके लिए रेखिक समीकरण निकाय

$$3x - y + 4z = 3,$$

$$x + 2y - 3z = -2,$$

$$6x + 5y + kz = -3,$$

has infinitely many solutions, is :

के अनन्त हल हैं, है :

(1) 3

(2) -5

(3) 5

(4) -3

Ans. (2)

Sol.  $\Delta = \begin{vmatrix} 3 & -1 & 4 \\ 1 & 2 & -3 \\ 6 & 5 & k \end{vmatrix}$

$$\Delta = 3(2k + 15) + 1(k + 18) + 4(5 - 12) = 0$$

$$\Rightarrow 7k + 35 = 0$$

$$\Rightarrow k = -5$$

17. If sum of the first 21 terms of the series  $\log_9 \frac{1}{2} x + \log_9 \frac{1}{3} x + \log_9 \frac{1}{4} x + \dots$ , where  $x > 0$  is 504, then  $x$  is equal to :

यदि श्रेणी  $\log_9 \frac{1}{2} x + \log_9 \frac{1}{3} x + \log_9 \frac{1}{4} x + \dots x > 0$  के प्रथम 21 पदों का योग 504 है, तो  $x$  बराबर है :

- (1) 243 (2) 7 (3) 81 (4) 9

Ans. (3)

Sol.  $2 \log_9 x + 3 \log_9 x + 4 \log_9 x \dots 21$  terms

$$= (2 + 3 + 4 + 5 + \dots + 22) \log_9 x = \frac{21}{2}(2+22) \log_9 x$$

$$= 21 \times 12 \log_9 x$$

$$= 252 \log_9 x = 504$$

$$\log_9 x = 2 \Rightarrow x = 81$$

18. For the natural numbers  $m, n$ , if  $(1 - y)^m (1 + y)^n = 1 + a_1 y + a_2 y^2 + \dots + a_{m+n} y^{m+n}$  and  $a_1 = a_2 = 10$ , then the value of  $(m + n)$  is equal to :

यदि धन पूर्णाकों  $m$  तथा  $n$  के लिए  $(1 - y)^m (1 + y)^n = 1 + a_1 y + a_2 y^2 + \dots + a_{m+n} y^{m+n}$  तथा  $a_1 = a_2 = 10$  हैं, तो  $(m + n)$  बराबर है :

- (1) 100 (2) 64 (3) 88 (4) 80

Ans. (4)

$$(1 - y)^m (1 + y)^n = 1 + a_1 y + a_2 y^2 + \dots$$

$$a_1 \text{ is coefficient of } y = 1. {}^n C_1 - {}^m C_1 = 10$$

$$\Rightarrow n - m = 10 \dots (1)$$

$$a_2 \text{ is coefficient of } y^2 = {}^m C_2 + {}^n C_2 = 10$$

$$\Rightarrow m(m-1) + n(n-1) - 2mn = 20 \dots (2)$$

$$m^2 + n^2 - 2mn - (m+n) = 20$$

$$(m-n)^2 - (m+n) = 20$$

$$(10)^2 - (m+n) = 20$$

$$(m+n) = 80$$

19. Consider the line  $L$  given by the equation  $\frac{x-3}{2} = \frac{y-1}{1} = \frac{z-2}{1}$ . Let  $Q$  be the mirror image of the point  $(2,3,-1)$  with respect to  $L$ . Let a plane  $P$  be such that it passes through  $Q$ , and the line  $L$  is perpendicular to  $P$ . Then which of the following points is on the plane  $P$  ?

माना  $Q$ , बिन्दु  $(2,3,-1)$  का रेखा  $L : \frac{x-3}{2} = \frac{y-1}{1} = \frac{z-2}{1}$  में दर्पण प्रतिबिम्ब है। माना एक समतल  $P$  बिन्दु  $Q$  से

होकर जाता है तथा रेखा  $L$ ,  $P$  पर लम्बवत् है। तो निम्न में से कौन सा बिन्दु समतल  $P$  पर है ?

- (1)  $(1, 1, 1)$  (2)  $(-1, 1, 2)$  (3)  $(1, 2, 2)$  (4)  $(1, 1, 2)$

Ans. (3)



**Sol.** Let A (2,3,-1)

Let image of A (2,3,-1) in the line mirror

$$\frac{x-3}{2} = \frac{y-1}{1} = \frac{z-2}{1} \text{ is } Q(\alpha, \beta, \gamma)$$

$$\Rightarrow \left( \frac{\alpha+2}{2}, \frac{\beta+3}{2}, \frac{\gamma-1}{2} \right) \text{ lies on } \frac{x-3}{2} = \frac{y-1}{1} = \frac{z-2}{1}$$

$$\Rightarrow \frac{\alpha-4}{4} = \frac{\beta+1}{2} = \frac{\gamma-5}{2} \dots\dots\dots(1)$$

Also  $AQ \perp$  to given line (L)

$$\Rightarrow 2(\alpha-2) + (\beta-3) + (\gamma+1) = 0$$

$$\Rightarrow 2\alpha + \beta + \gamma - 6 = 0 \dots\dots\dots(2)$$

by solving (1) and (2)

we get  $\alpha = 2, \beta = -2, \gamma = 4$

$$\Rightarrow Q(2, -2, 4)$$

Now equation of plane P which passes through Q(1,-1,5) and perpendicular to the line L is

$$2(x-2) + 1(y+2) + 1(z-4) = 0$$

$$2x+y+z = 6$$

Hence point (1,2,2) lies in it

**20.** Consider the following three statements :

(A) If  $3 + 3 = 7$  then  $4 + 3 = 8$ .

(B) If  $5 + 3 = 8$  then earth is flat.

(C) If both (A) and (B) are true then  $5 + 6 = 17$ .

Then which of the following statements is correct ?

(1) (A) and (C) are true while (B) is false

(2) (A) and (B) are false while (C) is true

(3) (A) is false, but (B) and (C) is true

(4) (A) is true while (B) and (C) are false

निम्न तीन कथनों पर विचार कीजिए :

(A) यदि  $3 + 3 = 7$  है, तो  $4 + 3 = 8$  है।

(B) यदि  $5 + 3 = 8$  है, तो पृथ्वी समतल है।

(C) यदि (A) तथा दोनों सत्य हैं, तो  $5 + 6 = 17$  है

तो निम्न में से कौन सा कथन सही है ?

(1) (A) और (C) सत्य हैं, जबकि (B) असत्य है

(2) (A) और (B) असत्य हैं, जबकि (C) सत्य है

(3) (A) असत्य है, परन्तु (B) तथा (C) सत्य हैं

(4) (A) सत्य है, जबकि (B) तथा (C) असत्य है

**Ans. (1)**

**Sol.** Truth table  $p \rightarrow q$

| p | q | $p \rightarrow q$ |
|---|---|-------------------|
| T | T | T                 |
| T | F | F                 |
| F | T | T                 |
| F | F | T                 |

A is true, B is false, C is true .

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## INTEGER QUESTIONS

1. Let a curve  $y = y(x)$  be given by the solution of the differential equation

$$\cos\left(\frac{1}{2}\cos^{-1}(e^{-x})\right)dx = \sqrt{e^{2x}-1}dy$$

If it intersects  $y$ -axis at  $y = -1$ , and the intersection point of the curve with  $x$ -axis is  $(\alpha, 0)$ , then  $e^\alpha$  is equal to \_\_\_\_\_.

माना वक्र  $y = y(x)$ , अवकल समीकरण  $\cos\left(\frac{1}{2}\cos^{-1}(e^{-x})\right)dx = \sqrt{e^{2x}-1}dy$  का हल है। यदि यह  $y$ -अक्ष को

$y = -1$  तथा  $x$ -अक्ष को  $(\alpha, 0)$  पर काटता है, तो  $e^\alpha$  बराबर है \_\_\_\_\_.

**Ans. (2)**

**Sol.**  $\cos\left(\frac{1}{2}\cos^{-1}(e^{-x})\right)dx = \sqrt{e^{2x}-1}dy$

$$\int \frac{\cos\left(\frac{1}{2}\cos^{-1}(e^{-x})\right)dx}{\sqrt{e^{2x}-1}} = \int dy$$

Put  $\cos^{-1}(e^{-x}) = t$

$$\frac{e^{-x}}{\sqrt{1-e^{-2x}}}dx = dt$$

$$\frac{dx}{\sqrt{e^{2x}-1}} = dt$$

$$\int \cos\left(\frac{t}{2}\right)dt = y + c$$

$$2\sin\left(\frac{1}{2}\cos^{-1}(e^{-x})\right) = y + c$$

At  $x=0 \Rightarrow y = -1$   
 $c = 1$

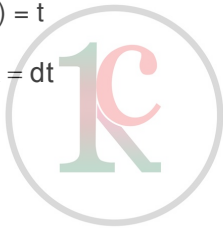
$$y = 2\sin\left(\frac{1}{2}\cos^{-1}(e^{-x})\right)$$

$y = 0$  then  $x = \alpha$

$$2\sin\left(\frac{1}{2}\cos^{-1}(e^{-x})\right) = 1$$

$$e^{-x} = \frac{1}{2} \Rightarrow x = \ln 2 = \alpha$$

So, the value of  $e^\alpha = 2$



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2. Let  $A = \{a_{ij}\}$  be a  $3 \times 3$  matrix, where

माना  $A = \{a_{ij}\}$  एक  $3 \times 3$  आव्यूह है, जबकि

$$a_{ij} = \begin{cases} (-1)^{j-i} & \text{if } i < j, \\ 2 & \text{if } i = j, \\ (-1)^{i+j} & \text{if } i > j, \end{cases}$$

then  $\det(3 \operatorname{Adj}(2A^{-1}))$  is equal to \_\_\_\_\_.

तो  $\det(3 \operatorname{Adj}(2A^{-1}))$  बराबर है \_\_\_\_\_.

**Ans. (108)**

**Sol.**  $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

So,  $|A| = \begin{vmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{vmatrix}$

$$= 2(4-1) + 1(-2+1) + 1(1-2)$$

$$= 2(3) + 1(-1) + 1(-1)$$

$$= 4$$

$$|3\operatorname{Adj}(2A^{-1})| = 3^3 |\operatorname{Adj}(2A^{-1})| = 3^3 \times |2A^{-1}|^2$$

$$= 3^3 \times 2^6 \times |A^{-1}|^2 = 3^3 \times 2^6 \times \frac{1}{|A|^2} = 108$$

3. The number of solutions of the equation  $\log_{(x+1)}(2x^2+7x+5) + \log_{(2x+5)}(x+1)^2 - 4 = 0$ ,  $x > 0$ , is \_\_\_\_\_.

समीकरण  $\log_{(x+1)}(2x^2+7x+5) + \log_{(2x+5)}(x+1)^2 - 4 = 0$ ,  $x > 0$ , के हलों की संख्या है \_\_\_\_\_.

**Ans. (1)**

**Sol.**  $\log_{(x+1)}((2x+5)(x+1)) + \log_{(2x+5)}(x+1)^2 = 4$

$$1 + \log_{(x+1)}(2x+5) + 2\log_{(2x+5)}(x+1) = 4$$

Put  $\log_{(x+1)}(2x+5) = t$

$$\therefore 1 + t + \frac{2}{t} = 4$$

$$t^2 + t + 2 = 4t \Rightarrow t^2 - 3t + 2 = 0$$

$$t = 1, t = 2$$

For  $t = 1$

$$2x + 5 = x + 1$$

$$\Rightarrow x = -4 \text{ (rejected)}$$

For  $t = 2$

$$2x + 5 = (x + 1)^2$$

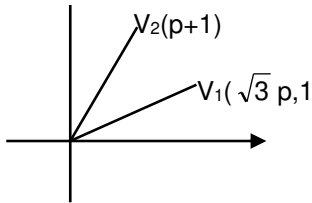
$$x = 2, x = -2 \text{ (rejected)}$$

4. For  $p > 0$ , a vector  $\vec{V}_2 = 2\hat{i} + (p+1)\hat{j}$  is obtained by rotating the vector  $\vec{V}_1 = \sqrt{3}p\hat{i} + \hat{j}$  by an angle  $\theta$  about origin in counter clockwise direction. If  $\tan\theta = \frac{(\alpha\sqrt{3} - 2)}{(4\sqrt{3} + 3)}$ , then the value of  $\alpha$  is equal to \_\_\_\_\_.

$p > 0$  के लिए, सदिश  $\vec{V}_1 = \sqrt{3}p\hat{i} + \hat{j}$  को केन्द्रबिन्दु के गिर्द वामावृत्त दिशा में कोण  $\theta$  से आवर्तित करने पर सदिश  $\vec{V}_2 = 2\hat{i} + (p+1)\hat{j}$  प्राप्त होता है। यदि  $\tan\theta = \frac{(\alpha\sqrt{3} - 2)}{(4\sqrt{3} + 3)}$  है, तो  $\alpha$  का मान बराबर है \_\_\_\_\_.

**Ans. (6)**

**Sol.**



$$|v_1| = |v_2|$$

$$\Rightarrow 3p^2 + 1 = 4 + (p+1)^2$$

$$\Rightarrow 2p^2 - 2p - 4 = 0$$

$$\Rightarrow p^2 - p - 2 = 0$$

$$\Rightarrow (p-2)(p+1) = 0$$

$$\Rightarrow p = -1, 2$$

$$\Rightarrow p = 2 \quad (p > 0)$$

If angle b/w  $\vec{v}_1$  &  $\vec{v}_2$  is  $\theta$

$$\Rightarrow \cos\theta = \frac{\vec{v}_1 \cdot \vec{v}_2}{|\vec{v}_1| |\vec{v}_2|} = \frac{(2\sqrt{3}\hat{i} + \hat{j}) \cdot (2\hat{i} + 3\hat{j})}{\sqrt{13}\sqrt{13}}$$

$$\Rightarrow \cos\theta = \frac{4\sqrt{3} + 3}{13}$$

$$\Rightarrow \tan\theta = \frac{\sqrt{112 - 24\sqrt{3}}}{4\sqrt{3} + 3}$$

$$\Rightarrow \tan\theta = \frac{2\sqrt{28 - 6\sqrt{3}}}{4\sqrt{3} + 3} = \frac{2(3\sqrt{3} - 1)}{4\sqrt{3} + 3}$$

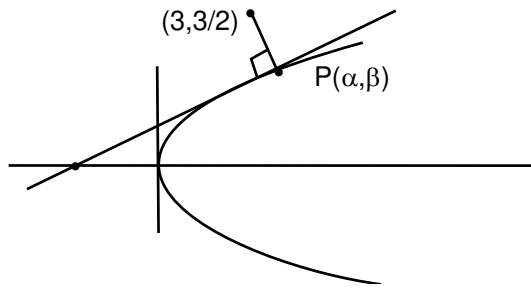
$$= \frac{6\sqrt{3} - 2}{4\sqrt{3} + 3} \Rightarrow \alpha = 6$$

5. If the point on the curve  $y^2 = 6x$ , nearest to the point  $\left(3, \frac{3}{2}\right)$  is  $(\alpha, \beta)$ , then  $2(\alpha + \beta)$  is equal to \_\_\_\_\_.

यदि वक्र  $y^2 = 6x$  पर, बिन्दु  $\left(3, \frac{3}{2}\right)$  के निकटतम बिन्दु  $(\alpha, \beta)$  है, तो  $2(\alpha + \beta)$  बराबर है \_\_\_\_\_.

**Ans. (9)**

Sol.



$$y^2 = 6x$$

$$2yy' = 6$$

$$\frac{dy}{dx} = \frac{3}{\beta}$$

$$-\frac{\beta}{3} = \frac{\beta - 3/2}{\alpha - 3}$$

$$-\frac{\beta}{3} = \frac{2\beta - 3}{2\alpha - 6}$$

$$-\beta(2\alpha - 6) = 6\beta - 9$$

$$6\beta - 2\alpha\beta = 6\beta - 9$$

$$\alpha\beta = \frac{9}{2} \Rightarrow \beta = \frac{9}{2\alpha}$$

$$\therefore \beta^2 = 6\alpha$$

$$\frac{81}{4\alpha^2} = 6\alpha$$

$$\alpha^3 = \frac{27}{8} \quad \alpha = \frac{3}{2}, \quad \beta^2 = 9 \Rightarrow \beta = \pm 3$$

$$\alpha = \frac{3}{2}, \quad \beta = 3$$

$$2(\alpha + \beta) = 9$$

6. Let  $\{a_n\}_{n=1}^{\infty}$  be a sequence such that  $a_1 = 1$ ,  $a_2 = 1$  and  $a_{n+2} = 2a_{n+1} + a_n$  for all  $n \geq 1$ . Then the value of

$$47 \sum_{n=1}^{\infty} \frac{a_n}{2^{3n}}$$
 is equal to \_\_\_\_\_.

माना  $\{a_n\}_{n=1}^{\infty}$  एक अनुक्रम है, जिसके लिए  $a_1 = 1$ ,  $a_2 = 1$  तथा  $a_{n+2} = 2a_{n+1} + a_n \forall n \geq 1$  है। तो  $47 \sum_{n=1}^{\infty} \frac{a_n}{2^{3n}}$  बराबर

है \_\_\_\_\_.

Ans. (7)

**Sol.** Let  $s = \sum_{n=1}^{\infty} \frac{a_n}{2^{3n}} = \sum_{n=1}^{\infty} \frac{a_n}{8^n} = \frac{a_1}{8^1} + \frac{a_2}{8^2} + \frac{a_3}{8^3} + \dots$  (i)

and  $a_{n+2} = 2a_{n+1} + a_n$

$$s = \sum_{n=1}^{\infty} \left( \frac{a_{n+2} - 2a_{n+1}}{8^n} \right)$$

$$= \sum_{n=1}^{\infty} \left( \frac{a_{n+2}}{8^n} - \frac{2a_{n+1}}{8^n} \right)$$

$$= \sum_{n=1}^{\infty} \left( \frac{8 \cdot a_{n+2}}{8^{n+2}} - \frac{2 \times 8 \times a_{n+1}}{8^{n+1}} \right)$$

$$s = 64 \sum_{n=1}^{\infty} \frac{a_{n+2}}{8^{n+2}} - 16 \sum_{n=1}^{\infty} \frac{a_{n+1}}{8^{n+1}}$$

$$s = 64 \left( \frac{a_3}{8^3} + \frac{a_4}{8^4} + \dots \right) - 16 \left( \frac{a_2}{8^2} + \frac{a_3}{8^3} + \dots \right)$$

$$\therefore s = 64 \left( s - \frac{a_1}{8} + \frac{a_2}{64} \right) - 16 \left( s - \frac{a_1}{8} \right), \text{ from (i)}$$

$$\therefore a_1 = a_2 = 1$$

$$\Rightarrow (47) s = 7$$

7. For  $k \in \mathbb{N}$ , let  $\frac{1}{\alpha(\alpha+1)(\alpha+2)\dots(\alpha+20)} = \sum_{k=0}^{20} \frac{A_k}{\alpha+k}$ , where  $\alpha > 0$ . Then the value of  $100 \left( \frac{A_{14} + A_{15}}{A_{13}} \right)^2$

is equal to \_\_\_\_\_.

माना  $k \in \mathbb{N}$  के लिए,  $\frac{1}{\alpha(\alpha+1)(\alpha+2)\dots(\alpha+20)} = \sum_{k=0}^{20} \frac{A_k}{\alpha+k}$ ,  $\alpha > 0$  है। तो  $100 \left( \frac{A_{14} + A_{15}}{A_{13}} \right)^2$  बराबर है \_\_\_\_\_.

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**Ans.** (9)

**Sol.**  $\frac{1}{\alpha(\alpha+1)(\alpha+2)\dots(\alpha+20)} = \sum_{k=0}^{20} \frac{A_k}{\alpha+k}$

$$\Rightarrow \frac{1}{\alpha(\alpha+1)(\alpha+2)\dots(\alpha+20)} = \frac{A_0}{\alpha} + \frac{A_1}{\alpha+1} + \frac{A_2}{\alpha+2} + \frac{A_3}{\alpha+3} + \dots + \frac{A_{20}}{\alpha+20}$$

$$\Rightarrow 1 = A_0(\alpha+1)(\alpha+2)\dots(\alpha+20) + A_1(\alpha)(\alpha+2)(\alpha+3)\dots(\alpha+20) + \dots + A_{20}\alpha(\alpha+1)(\alpha+2)\dots(\alpha+19)$$

$$\Rightarrow \text{Put } \alpha = -14 \Rightarrow A_{14} = \frac{1}{(-14)(-13)(-12)\dots(-1)(1)(2)\dots(6)}$$

$$A_{15} = \frac{1}{(-15)(-14)(-13)\dots(-1)(1)(2)\dots(5)}$$

$$A_{13} = \frac{1}{(-13)(-12)\dots(-1)(1)(2)\dots(7)}$$

$$\begin{aligned} &\Rightarrow \left( \frac{A_{15} + A_{14}}{A_{13}} \right)^2 = \left( \frac{A_{15}}{A_{13}} + \frac{A_{14}}{A_{13}} \right)^2 \\ &= \left( \frac{6.7}{15.14} - \frac{7}{14} \right)^2 \\ &= \left( \frac{1}{5} - \frac{1}{2} \right)^2 \frac{9}{100} \\ &\Rightarrow 100 \left( \frac{A_{15} + A_{14}}{A_{13}} \right)^2 = 9 \end{aligned}$$

8. Consider a triangle having vertices A (-2, 3), B(1, 9) and C(3, 8). If a line L passing through the circumcenter of triangle ABC, bisects line BC, and intersects y-axis at point  $\left(0, \frac{\alpha}{2}\right)$ , then the value of real number  $\alpha$  is \_\_\_\_\_.

एक त्रिभुज जिसके शीर्ष बिन्दु A (-2, 3), B(1, 9) तथा C(3, 8) हैं, पर विचार कीजिए। यदि एक रेखा L, जो त्रिभुज ABC के परिकेन्द्र से होकर जाती है रेखा BC को समद्विभाजित करती है तथा y- अक्ष को बिन्दु  $\left(0, \frac{\alpha}{2}\right)$  पर काटती है, तो

वास्तविक संख्या  $\alpha$  का मान है \_\_\_\_\_.

Ans. (9)

Mid point of BC is  $\left(2, \frac{17}{2}\right)$  and slope of BC is  $\left(-\frac{1}{2}\right)$ .

Since a line passes through circumcentre of  $\Delta ABC$  and bisects the side BC is perpendicular bisector of side BC.

Equation of required line is

$$y - \frac{17}{2} = 2(x - 2)$$

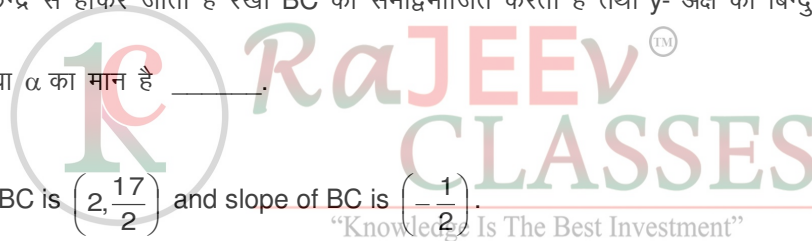
$$2x - y = 4 - \frac{17}{2}$$

$$4x - 2y + 9 = 0$$

It intersects the y-axis at  $\left(0, \frac{\alpha}{2}\right)$

$$\Rightarrow -\alpha + 9 = 0$$

$$\Rightarrow \alpha = 9$$



9. Let a function  $g : [0,4] \rightarrow \mathbb{R}$  be defined as  $g(x) = \begin{cases} \max \{t^3 - 6t^2 + 9t - 3\}, 0 \leq x \leq 3 \\ 0 \leq t \leq x \\ 4 - x, 3 < x \leq 4 \end{cases}$  then the number of points in the interval  $(0, 4)$  where  $g(x)$  is NOT differentiable, is \_\_\_\_\_.

माना एक फलन  $g : [0,4] \rightarrow \mathbb{R}$ ,  $g(x) = \begin{cases} \max \{t^3 - 6t^2 + 9t - 3\}, 0 \leq x \leq 3 \\ 0 \leq t \leq x \\ 4 - x, 3 < x \leq 4 \end{cases}$  द्वारा परिभाषित है, तो अन्तराल  $(0, 4)$

में जिन बिन्दुओं पर  $g(x)$  अवकलनीय नहीं है, उनकी संख्या है \_\_\_\_\_.

**Ans.**

(1)

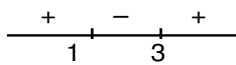
**Sol.**

$$f(t) = t^3 - 6t^2 + 9t - 3$$

$$f'(t) = 3t^2 - 12t + 9 = 0$$

$$= t^2 - 4t + 3 = 0$$

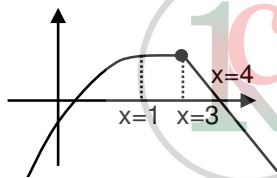
$$t = 1, 3$$



$$f(1) = 1$$

$$f(3) = -3$$

$$g(x) = \begin{cases} x^3 - 6x^2 + 9x - 3 & 0 \leq x \leq 1 \\ 1 & 1 \leq x \leq 3 \\ 4 - x & 3 < x \leq 4 \end{cases}$$



Function is non differentiable at  $x = 3$  "Knowledge Is The Best Investment"

10. If  $\lim_{x \rightarrow 0} \frac{\alpha x e^x - \beta \log_e(1+x) + \gamma x^2 e^{-x}}{x \sin^2 x} = 10$ ,  $\alpha, \beta, \gamma \in \mathbb{R}$ , then value of  $\alpha + \beta + \gamma$  is \_\_\_\_\_.

यदि  $\lim_{x \rightarrow 0} \frac{\alpha x e^x - \beta \log_e(1+x) + \gamma x^2 e^{-x}}{x \sin^2 x} = 10$ ,  $\alpha, \beta, \gamma \in \mathbb{R}$  है, तो  $\alpha + \beta + \gamma$  का मान है \_\_\_\_\_.

**Ans.** (3)

$$\lim_{x \rightarrow 0} \frac{\alpha x \left( 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \right) - \beta \left( x - \frac{x^2}{2} + \frac{x^3}{3} + \dots \right) + \gamma x^2 \left( 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \dots \right)}{x^3} = 10$$

$$\Rightarrow \alpha - \beta = 0, \Rightarrow \alpha = \beta$$

$$\Rightarrow \alpha + \frac{\beta}{2} + \gamma = 0 \Rightarrow \gamma = -\frac{3\beta}{2}$$

$$\Rightarrow \frac{\alpha}{2} - \frac{\beta}{3} - \gamma = 10 \Rightarrow \frac{\beta}{2} - \frac{\beta}{3} + \frac{3\beta}{2} = 10 \Rightarrow \frac{3\beta - 2\beta + 9\beta}{6} = 10$$

$$\therefore \beta = 6, \alpha = 6, \gamma = -9$$

So, the value of  $\alpha + \beta + \gamma = 3$