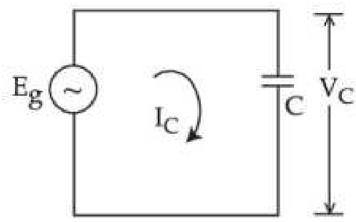
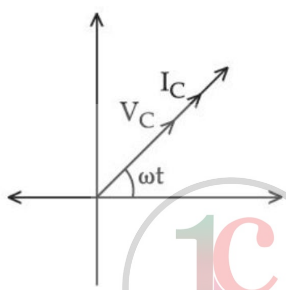
Single Choice Type

This section contains **20 Single choice questions.** Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which **Only One** is correct.

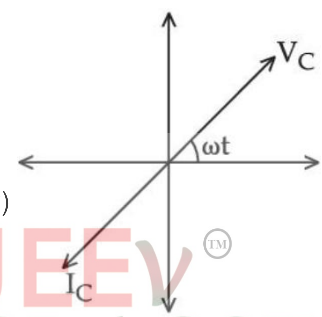
1. In a circuit consisting of a capacitance and a generator with alternating emf $E_g = E_{g_0} \sin\omega t$ V_C and I_C are the voltage and current. Correct phasor diagram for such circuit is :



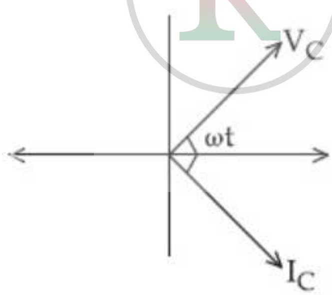
(1)



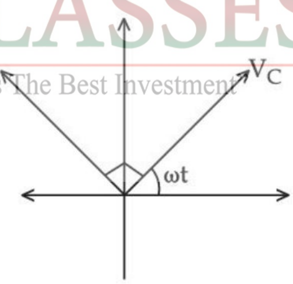
(2)



(3)



(4)



Ans. **(4)**

Sol. In pure capacitive circuit, current leads voltage by $\pi/2$ phase.

2. Intensity of sunlight is observed as 0.092 Wm^{-2} at a point in free space. What will be the peak value of magnetic field at that point ? ($\varepsilon_0 = 8.85 \times 10^{-12} C^2N^{-1}m^{-2}$).

(1) 1.96×10^{-8} T (2) 5.88 T (3) 2.77×10^{-8} T (4) 8.31 T

Ans. **(3)**

Sol. $\frac{1}{2}\varepsilon_0 E_0^2 . C = I$

$E_0^2 = \frac{2I}{\varepsilon_0 C}$ $E_0 = \sqrt{\frac{2I}{\varepsilon_0 C}}$

$B_0 = \frac{E_0}{C} = \frac{1}{C}\sqrt{\frac{2I}{\varepsilon_0 C}} = 2.77 \times 10^{-8}$ T

3. Choose the correct option :

(1) True dip is always equal to apparent dip.

(2) True dip is less than the apparent dip.

(3) True dip is always greater than the apparent dip.

(4) True dip is not mathematically related to apparent dip.

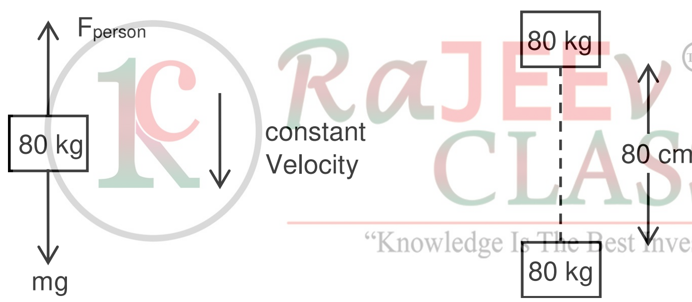
Ans. **(2)**

4. A porter lifts a suitcase of mass 80 kg and at the destination lowers it down by a distance of 80 cm with a constant velocity. Calculate the work done by the porter in lowering the suitcase. (take g = 9.8 ms^{-2})

(1) 784.0 J (2) – 62700.0 J (3) 627.2 J (4) – 627.2 J

Ans. **(4)**

Sol.



$F_{person} = mg = 784$ N

$W_{person} = F_p s \cos 180° = -784 \times 80 \times 10^{-2} = -627.2$ J

5. T_0 is the time period of a simple pendulum at a place. If the length of the pendulum is reduced to $\frac{1}{16}$ times of its initial value, the modified time period is :

(1) T_0 (2) $\frac{1}{4}T_0$ (3) $8\pi T_0$ (4) $4T_0$

Ans. **(2)**

Sol. $T = 2\pi\sqrt{\frac{\ell}{g}}$

$T' = 2\pi\sqrt{\frac{\ell}{16g}} = \frac{T}{4}$

6. Consider a situation in which a ring, a solid cylinder and a solid sphere roll down on the same inclined plane without slipping, Assume that they start rolling from rest and having identical diameter. The correct statement for this situation is :

(1) The sphere has the greatest and the ring has least velocity of the centre of mass at the bottom of the inclined plane.

(2) The ring has the greatest and the cylinder has the least velocity of the centre of mass at the bottom of the inclined plane.

(3) All of them will have same velocity

(4) The cylinder has the greatest and the sphere has the least velocity of the centre of mass at the bottom of the inclined plane.

Ans. **(1)**

Sol. $mgh = \frac{1}{2}I_{cm}\omega^2 + \frac{1}{2}mv^2$

$$V = \sqrt{\frac{2gh}{1+\frac{I_{cm}}{mR^2}}}$$

As $I\uparrow$, $V\downarrow$

7. What will be the average value for a monoatomic gas in thermal equilibrium at temperature T?

(1) $\frac{1}{2}k_BT$ (2) $\frac{3}{2}k_BT$ (3) $\frac{2}{3}k_BT$ (4) k_BT

Ans. **(2)**

8. Match List-I with List-II :

List-I	List-II
(a) $\omega L > \frac{1}{\omega C}$	(i) Current is in phase with emf
(b) $\omega L = \frac{1}{\omega C}$	(ii) Current lags behind the applied emf
(c) $\omega L < \frac{1}{\omega C}$	(iii) Maximum current occurs
(d) Resonant frequency	(iv) Current leads the emf

Choose the correct answer from the option given below:

(1) (a)-(iv); (b)-(iii); (c)-(ii); (d)-(i)
(2) (a)-(ii); (b)-(i); (c)-(iv); (d)-(iii)
(3) (a)-(ii); (b)-(i); (c)-(iii); (d)-(iv)
(4) (a)-(iii); (b)-(i); (c)-(iv); (d)-(ii)

Ans. **(2)**

9. Consider a situation in which reverse biased current of a particular P-N junction increases when it is exposed to a light of wavelength ≤ 621 nm. During this process, enhancement in carrier concentration takes place due to generation of hole-electron pairs. The value of band gap is nearly.

(1) 4eV (2) 1 eV (3) 2 eV (4) 0.5 eV

Ans. **(3)**

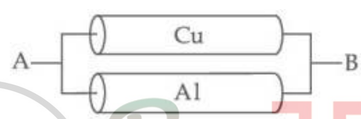
Sol. $E = \frac{hc}{\lambda} = eV$

$V = \frac{hc}{\lambda e}$

$= \frac{1224 \text{ ev} - \text{nm}}{e \times 612 \text{nm}} = 2 \text{ volt}$

Band gap = 2eV

10. A copper (Cu) rod of length 25 cm and cross-sectional area 3mm^2 is joined with a similar Aluminium (Al) rod as shown in figure. Find the resistance of the combination between the ends A and B. (Take Resistivity of Copper = 1.7×10^{-8} Ωm)



(1) 0.858 mΩ (2) 0.0858 mΩ (3) 2.170 mΩ (4) 1.420 mΩ

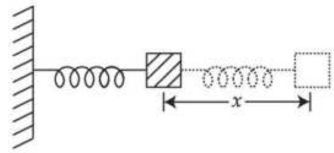
Ans. **(1)**

Sol. $R_{eq} = \frac{R_1R_2}{R_1 + R_2} = \frac{\rho_1 \frac{\ell}{A} \rho_2\left(\frac{\ell}{A}\right)}{\rho_1\left(\frac{\ell}{A}\right) + \rho_2\left(\frac{\ell}{A}\right)}$

$= \frac{\ell}{A}\left(\frac{\rho_1\rho_2}{\rho_1 + \rho_2}\right) = \frac{0.25}{3\times10^{-6}}\left(\frac{1.7\times10^{-8}\times2.6\times10^{-8}}{1.7\times10^{-8}+2.6\times10^{-8}}\right)$

$= 0.085 \times 10^{-2} = 0.858$ mΩ

11. The motion of a mass on a spring, with spring constant K is as shown in figure.



The equation of motion is given by x(t) = Asinωt + Bcosωt $\omega = \sqrt{\frac{K}{m}}$

Supper that at time t = 0, the position of mass is x(0) and velocity υ(0), then its displacement can also be represented as x(t) = Ccos(ωt – φ), where C and φ are :

(1) $C = \sqrt{\frac{\upsilon(0)^2}{\omega^2} + x(0)^2}, \phi = \tan^{-1}\left(\frac{\upsilon(0)}{x(0)\omega}\right)$

(2) $C = \sqrt{\frac{2\upsilon(0)^2}{\omega^2} + x(0)^2}, \phi = \tan^{-1}\left(\frac{x(0)\omega}{2\upsilon(0)}\right)$

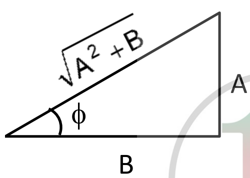
(3) $C = \sqrt{\frac{2\upsilon(0)^2}{\omega^2} + x(0)^2}, \phi = \tan^{-1}\left(\frac{\upsilon(0)}{x(0)\omega}\right)$

(4) $C = \sqrt{\frac{\upsilon(0)^2}{\omega^2} + x(0)^2}, \phi = \tan^{-1}\left(\frac{x(0)\omega}{\upsilon(0)}\right)$

Ans. **(1)**

Sol. x(t) = Asinωt + Bcosωt v(t) = Aωcosωt – Bωsinωt

x(0) = B , v(0) = Aω



$\sqrt{A^2 + B^2}\cos(\omega t - \phi)$

$C = \sqrt{A^2 + B^2} \Rightarrow C = \sqrt{\frac{\upsilon(0)^2}{\omega^2} + x(0)^2}$

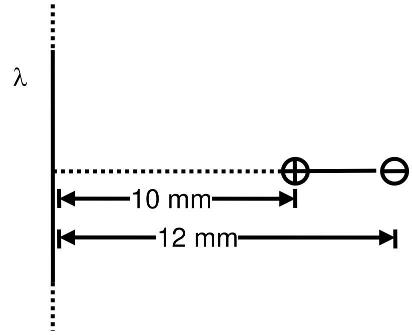
$\phi = \tan^{-1}\left(\frac{A}{B}\right) \Rightarrow \phi = \tan^{-1}\left(\frac{\upsilon(0)}{x(0)\omega}\right)$

12. An electric dipole is placed on x-axis in proximity to a line charge of linear charge density 3.0×10^{-6} C/m. Line charge is placed on z-axis and positive and negative charge of dipole is at a distance of 10 mm and 12 mm from the origin respectively. If total force of 4 N is exerted on the dipole, find out the amount of positive or negative charge of the dipole.

(1) 0.485 mC (2) 4.44 μC (3) 815.1 nC (4) 8.8 μC

Ans. **(2)**

Sol.



Let charge be Q

Net force $= \dfrac{2k\lambda Q}{r_1} + \dfrac{2k\lambda(-Q)}{r_2}$

$4N = 2k\lambda Q\left[\dfrac{1}{r_1} - \dfrac{1}{r_2}\right]$

$4N = 2 \times 9 \times 10^9 \times 3 \times 10^{-6}\, Q\left[\dfrac{1}{10} - \dfrac{1}{12}\right] \times 10^3$

$4N = 54 \times 10^6\, Q \times \dfrac{1}{60}$

$Q = 4.44 \times 10^{-6}\, C = 4.44\, \mu C$

13. What will be the projection of vector $\vec{A} = \hat{i} + \hat{j} + \hat{k}$ on vector $\vec{B} = \hat{i} + \hat{j}$?

(1) $2(\hat{i} + \hat{j} + \hat{k})$ (2) $\sqrt{2}(\hat{i} + \hat{j})$ (3) $(\hat{i} + \hat{j})$ (4) $\sqrt{2}(\hat{i} + \hat{j} + \hat{k})$

Ans. **(3)**

Sol. $|\vec{A}|\cos\theta\hat{B}$

$\dfrac{\vec{A}.\vec{B}}{|\vec{B}|}\hat{B} = \hat{i} + \hat{j}$

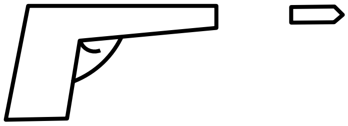
14. A bullet of '4 g' mass is fired from a gun of mass 4 kg. If the bullet moves the muzzle speed of 50 ms^{-1}, the impulse imparted to the gun and velocity of recoil of gun are :

(1) 0.4 kg ms^{-1}, 0.1 ms^{-1} (2) 0.2 kg ms^{-1}, 0.1 ms^{-1}

(3) 0.2 kg ms^{-1}, 0.05 ms^{-1} (4) 0.4 kg ms^{-1}, 0.05 ms^{-1}

Ans. **(3)**

Sol.



Initial momentum = final momentum

$0 = m_G V_G + M_B V_b$

$\Rightarrow \quad 0 = 4 \times V_G + \frac{4}{1000} V_b$

$\Rightarrow \quad V_b = -1000\ V_G$ ....(1)

$V_{bG} = V_b - V_G$

$\Rightarrow \quad 50 = V_B - V_G$ ....(2)

$\Rightarrow \quad 50 = -1001\ V_G$

$V_G \approx 0.05$ m/s

impulse = $m_G V_G = 4 \times 0.05 = 0.2$ kg m/s

15. **Statement-I:** The ferromagnetic property depends on temperature. At high temperature, ferromagnet becomes paramagnet.

Statement-II: At high temperature, the domain wall area of a ferromagnetic substance increases.

In the light of the above statements, choose the most appropriate answer from the option given below:

(1) Both Statement-I and Statement-II are true

(2) Statement-I is false but Statement-II is true

(3) Both Statement-I and Statement-II are false

(4) Statement-I is true but Statement-II is false

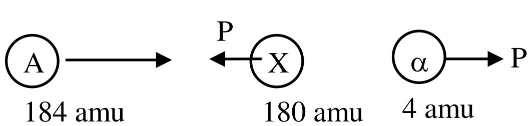
Ans. (4)

16. A nucleus with mass number 184 initially at rest emits an α-particle. If the Q value of the reaction is 5.5 MeV, calculate the kinetic energy of the α-particle.

(1) 5.5 MeV (2) 5.0 MeV (3) 5.38 MeV (4) 0.12 MeV

Ans. (3)

Sol.



$$k_\alpha = \left(\frac{A-4}{A}\right)Q = 5.38 \text{ MeV}$$

17. What should be the height of transmitting antenna and the population covered if the television telecast is to cover a radius of 150 km? The average population density around the tower is 2000/ km^2 and the value of $R_e = 6.5 \times 10^6$ m.

(1) Height = 1731 m, Population Covered = 1413×10^5

(2) Height = 1600 m, Population Covered = 2×10^5

(3) Height = 1800 m, Population Covered = 1413×10^8

(4) Height = 1241 m, Population Covered = 7×10^5

Ans. **(1)**

Sol. Radius of earth = 6400 km

d = 150 km

height of Antena = ?

$d = \sqrt{2Rh}$

$h = \dfrac{d^2}{2R} = \dfrac{150 \times 150 \times 10^6}{2 \times 6.5 \times 10^6} = 1730.7 \approx 1731$ m

Population covered $\Rightarrow 2\pi Rh \times$ density

$2\pi \times 6.5 \times 10^6 \times 1730.7 \times 2000 \times 10^{-6} \approx 1413 \times 10^5$

18. An electron of mass m_e and a proton of mass m_p are accelerated through the same potential difference. The ratio of the de-Broglie wavelength associated with the electron to that with the proton is:

(1) $\dfrac{m_p}{m_e}$ (2) 1 (3) $\dfrac{m_e}{m_p}$ (4) $\sqrt{\dfrac{m_p}{m_e}}$

Ans. **(4)**

Sol. $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2mqV}}$

$\dfrac{\lambda_e}{\lambda_\rho} = \sqrt{\dfrac{m_\rho}{m_e}}$

19. A body is projected vertically upwards from the surface of earth with a velocity sufficient enough to carry to infinity. The time taken by it to reach height h is______s.

(1) $\sqrt{\dfrac{2R_e}{g}}\left[\left(1+\dfrac{h}{R_e}\right)^{\frac{3}{2}} - 1\right]$

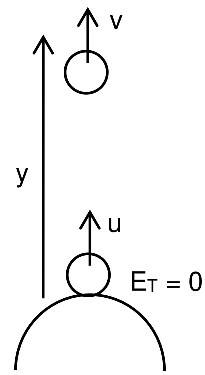
(2) $\sqrt{\dfrac{R_e}{2g}}\left[\left(1+\dfrac{h}{R_e}\right)^{\frac{3}{2}} - 1\right]$

(3) $\dfrac{1}{3}\sqrt{\dfrac{R_e}{2g}}\left[\left(1+\dfrac{h}{R_e}\right)^{\frac{3}{2}} - 1\right]$

(4) $\dfrac{1}{3}\sqrt{\dfrac{2R_e}{g}}\left[\left(1+\dfrac{h}{R_e}\right)^{\frac{3}{2}} - 1\right]$

Ans. (4)

Sol.



$$\frac{1}{2}mv^2 - \frac{GMm}{R+y} = 0$$

$$\Rightarrow v = \sqrt{\frac{2GM}{R+y}}$$

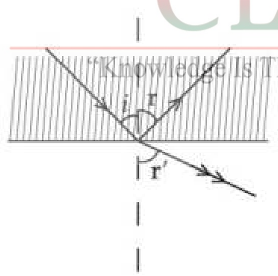
$$\Rightarrow \frac{dy}{dt} = \sqrt{\frac{2Gm}{R+y}}$$

$$\Rightarrow \int_0^y \sqrt{R+y}\,dy = \sqrt{2Gm}\int_0^t dt \qquad \Rightarrow \left[\frac{2(R+y)^{\frac{3}{2}}}{3}\right]_0^y = \sqrt{2Gm}\ t$$

$$\Rightarrow \frac{2}{3}\left[\left((R+y)^{\frac{3}{2}} - R^{\frac{3}{2}}\right)\right] = \sqrt{2Gm}\,.t \qquad \Rightarrow t = \frac{\frac{2}{3}\left[(R+y)^{\frac{3}{2}} - R^{\frac{3}{2}}\right]}{\sqrt{2Gm}}$$

$$\Rightarrow \frac{1}{3}\sqrt{\frac{2R_e}{g}}\left[\left(1+\frac{h}{R_e}\right)^{\frac{3}{2}} - 1\right]$$

20. A ray of light passes from a denser medium to a rarer medium at an angle of incidence i. The reflected and refracted rays make an angle of 90° with each other. The angle of reflection and refraction are respectively r and r'. The critical angle is given by:



(1) $\sin^{-1}(\tan r)$ (2) $\sin^{-1}(\cot r)$ (3) $\tan^{-1}(\sin i)$ (4) $\sin^{-1}(\tan r')$

Ans. (1)

Sol. $i = r$ & $r' = 90 - r$

$$\theta_C = \sin^{-1}\left(\frac{1}{\mu}\right)$$

$$= \sin^{-1}\left(\frac{\sin i}{\sin r'}\right) = \sin^{-1}\left(\frac{\sin i}{\sin r}\right) = \sin^{-1}(\tan r)$$

Numerical Value Type

This section contains **10 Numerical value type questions.**

1. The area of cross-section of a railway track is 0.01 m^2. The temperature variation is 10°C. Coefficient of linear expansion of material of track 10^{-5}/°C. The energy stored per meter in the track is____J/m. (Young's modulus of material of trak is 10^{11} Nm^{-2})

Ans. 5

Sol. $U = \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume}$

$U = \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume} \times A\ell$

$\frac{U}{\ell} = \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume} \times A$

$= \frac{1}{2} \times Y \times (\text{strain})^2 \times A$

$= \frac{1}{2} \times Y\left(\frac{\Delta\ell}{\ell}\right)^2 \times A = \frac{1}{2}Y\left(\frac{\ell\alpha\Delta t}{\ell}\right)^2 \times A = \frac{1}{2} Y\alpha^2 \Delta t^2 A$

$= \frac{1}{2} \times 10^{11} \times 10^{-10} \times 10 \times 10 \times 10^{-2} = 5$ Joule/m

2. In 5 minutes, a body cools from 75°C to 65°C at room temperature of 25°C. The temperature of body at the end of next 5 minutes is _____°C.

Ans. 57

Sol. $\frac{\Delta T}{t} = K\left(\frac{T_1 + T_2}{2} - T_0\right)$

$\frac{75-65}{5} = K\left(\frac{75+65}{2} - 25\right)$ ...(1)

$\frac{65-T}{5} = K\left(\frac{T+65}{2} - 25\right)$ ...(2)

Eq(2)/Eq(1)

$\frac{65-T}{75-65} = \frac{\frac{T+65}{2} - 25}{\frac{75+65}{2} - 25}$

$\frac{65-T}{10} = \frac{T+15}{90}$

$90 \times 65 - 90\,T = 10\,T + 10 \times 15$

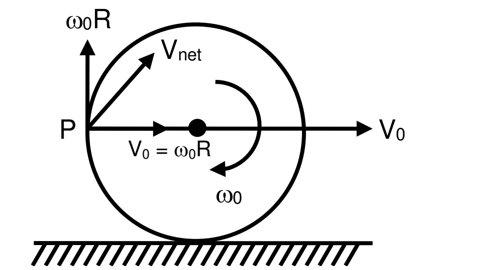
$100\,T = 90 \times 65 - 15 \times 10$

$T = 57$°C

3. The centre of a wheel rolling on a plane surface moves with a speed υ_0. A particle on the rim of the wheel at the same level as the centre will be moving at speed $\sqrt{x}\upsilon_0$. Then the value of x is____.

Ans. 2

Sol.

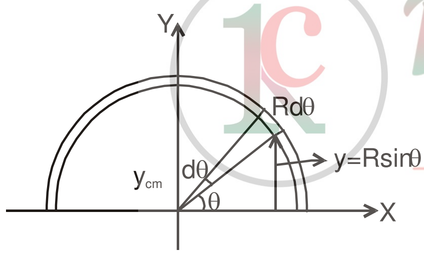


$$(V_p)_{net} = \sqrt{V_0^2 + (\omega_0 R)^2} = \sqrt{V_0^2 + V_0^2} = \sqrt{2}V_0$$

4. The position of the centre of mass of a uniform semi-circular wire of radius 'R' placed in x-y plane with its centre at the origin and the line joining its ends as x-axis is given by $\left(0, \frac{xR}{\pi}\right)$. Then, the value of $|x|$ is___.

Ans. 2

Sol.



To find y_{cm} we use $y_{cm} = \frac{1}{M}\int dmy$ .....(i)

Here for dm we consider an elemental arc of the ring at an angle θ from the x-direction of angular width dθ. If radius of the ring is R then its y coordinate will be R sinθ, here dm is given as

$$dm = \frac{M}{\pi R} \times R\,d\theta$$

So from equation ....(i), we have

$$y_{cm} = \frac{1}{M}\int_0^{\pi} \frac{M}{\pi R} R d\theta\ (R \sin\theta) = \frac{R}{\pi}\int_0^{\pi} \sin\theta d\theta$$

$$y_{cm} = \frac{2R}{\pi} \quad .....(ii)$$

$\therefore$ x = 2

5. Three particles P, Q and R moving along the vectors $\vec{A} = \hat{i} + \hat{j}$, $\vec{B} = \hat{j} + \hat{k}$ and $\vec{C} = -\hat{i} + \hat{j}$ respectively. They strike on a point and start to move in different directions. Now particle P is moving normal to the plane which contains vector $\vec{A}$ and $\vec{B}$. Similarly particle Q is moving normal to the plane which contains vector $\vec{A}$ and $\vec{C}$. The angle between the direction of motion of P and Q is $\cos^{-1}\left(\frac{1}{\sqrt{x}}\right)$. Then the value of x is_____.

Ans. **3**

Sol. $\vec{P} = K(\vec{A} \times \vec{B})$

$$\begin{bmatrix} i & j & k \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$= K(\hat{i} - \hat{j} + \hat{k})$

$\vec{Q} = l(\vec{A} \times \vec{C})$

$$\begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ -1 & 1 & 0 \end{bmatrix} = 2(\hat{k})l$$

$$\cos\theta = \frac{\vec{P} \bullet \vec{Q}}{|\vec{P}||\vec{Q}|} = \frac{1}{\sqrt{3}}$$

$$\theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

6. The total charge enclosed in an incremental volume of 2×10^{-9} m^3 located at the origin is ____nC, if electric flux density of its field is found as $D = e^{-x}\sin y\hat{i} - e^{-x}\cos y\hat{j} + 2z\hat{k}$ C/m^2.

Ans. $4\ \varepsilon_0$

Sol. $\vec{\nabla}.\vec{E} = \frac{\rho}{\varepsilon_0}$

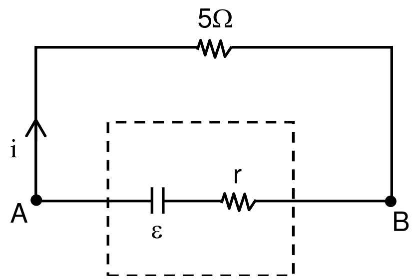
$$\frac{\delta E}{\delta x} + \frac{\delta E}{\delta y} + \frac{\delta E}{\delta z} = \frac{\rho}{\varepsilon_0}$$

$\rho = 2\varepsilon_0 \Rightarrow Q = 4\varepsilon_0$

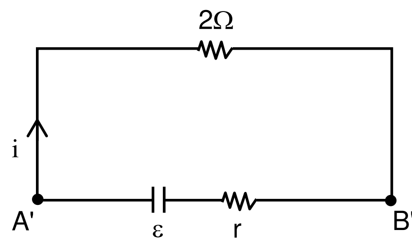
7. In and electric circuit, a cell of certain emf provides a potential difference of 1.25 V across a load resistance of 5 Ω. However, it provides a potential difference of 1V across a load resistance of 2 Ω. The emf of the cell is given by $\frac{x}{10}$ V . Then the value of x is_____.

Ans. 15

Sol.



(1)



(2)

(1) $I = \frac{1.25}{5} = 0.25$ A

$V_{AB} = 5V = \varepsilon - ir$

$1.25 = \varepsilon - 0.25\ r$ ...(1)

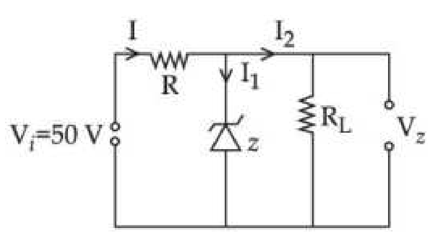
(2) $I = \frac{1}{2} = 0.5$ A

$V_{A'B'} = 1 = \varepsilon - 0.5r$ ...(2)

Solving (1) & (2)

$\varepsilon = 15$ v

8. In a given circuit diagram, a 5 V zener diode along with a series resistance is connected across a 50 V power supply. The minimum value of the resistance required, if the maximum zener current is 90 mA will be_____Ω.



Ans. **500**

Sol. $\frac{45}{R} \geq 90mA$

$R \leq \frac{45}{90} \times 10^3$

$R \leq 500\ \Omega$

9. Three students S_1, S_2 and S_3 perform an experiment for determining the acceleration due to gravity (g) using a simple pendulum. They use different lengths of pendulum and record time for different number of oscillations. The observations are shown in the table.

Student No.	Length of Pendulum (cm)	No. of oscillations (n)	Total time for n oscillations	Time period (s)
1	64.0	8	128.0	16.0
2	64.0	4	64.0	16.0
3	20.0	4	36.0	9.0

(Least count of length = 0.1 cm

least count for time = 0.1 s)

If E_1, E_2 and E_3 are the percentage errors in 'g' for students 1, 2 and 3 respectively, then the minimum percentage error is obtained by student no.______.

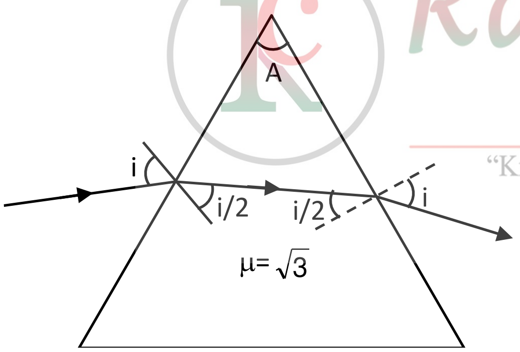
Ans. **1**

Sol. Percentage error in 'g' = $\left(\frac{\Delta\ell}{\ell}+\frac{2\Delta T}{T}\right)\times 100$

10. A ray of light passing through a prism ($\mu = \sqrt{3}$) suffers minimum deviation. It is found that the angle of incidence is double the angle of refraction within the prism. Then, the angle of prism is_____(in degree).

Ans. **60**

Sol.



$A = \frac{i}{2}+\frac{i}{2} = i$

$1\sin i = \mu\sin\frac{i}{2}$

$2\sin\frac{i}{2}\cos\frac{i}{2} = \sqrt{3}\sin\frac{i}{2}$

$\cos\frac{i}{2} = \frac{\sqrt{3}}{2}$

$\frac{i}{2} = 30°$

$i = 60°$

$\therefore \quad A = i = 60°$

CHEMISTRY

Section : Chemistry Section A

1. Which one of the following molecules does not show stereo isomerism?

(1) 3,4-Dimethylhex-3-ene (2) 3-Ethylhex-3-ene

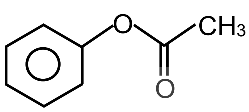
(3) 3-Methylhex-1-ene (4) 4-Methylhex-1-ene

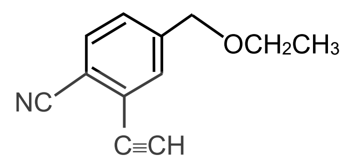
Ans. **(2)**

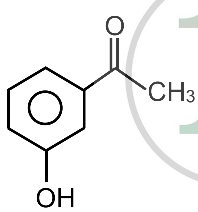
Sol. $CH_3–CH_2–C(CH_2–CH_3)=CH–CH_2–CH_3$

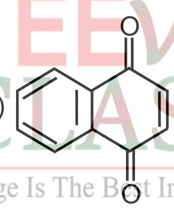
Neither show geometrical nor show optical isomerism.

2. Which one of the following compounds will provide a tertiary alcohol on reaction with excess of CH_3MgBr followed by hydrolysis?

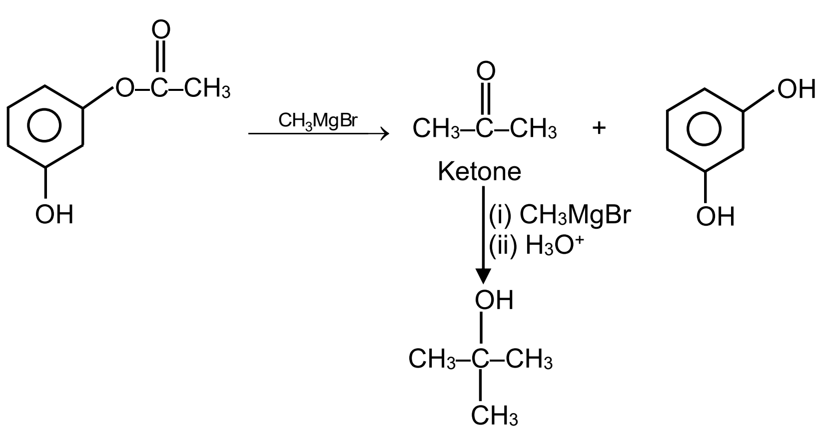
(1) 

(2) 

(3) 

(4) 

Ans. **(1)**

Sol. 

(tertiory Alcohol)

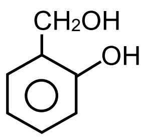
3. When $AgNO_3$ solution is added to KI solution then the sol produced is

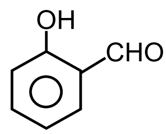
(1) AgI/Ag^+ (2) KI/NO_3^- (3) $AgNO_3/NO_3^-$ (4) AgI/I^-

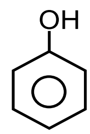
Ans. **(4)**

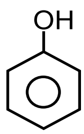
Sol. $AgNO_3 + KI \longrightarrow AgI/I^-$

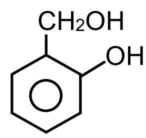
4. An organic compound $A(C_6H_6O)$ gives dark green colouration with ferric chloride. On treatment with $CHCl_3$. and KOH followed by acidification gives compound B. Compound B can also be obtained from compound C on reaction with pyridinium chlorochromate (PCC).

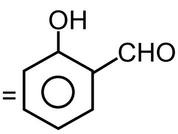
(1) A = 

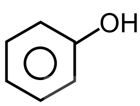
B = 

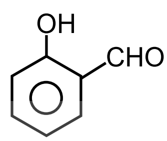
C = 

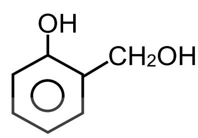
(2) A = 

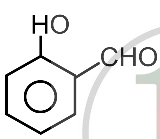
B = 

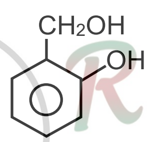
C = 

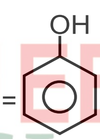
(3) A = 

B = 

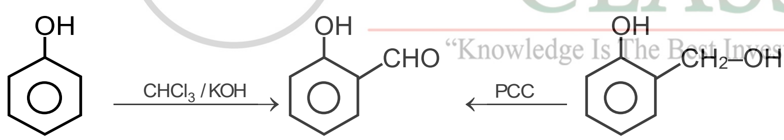
C = 

(4) A = 

B = 

C = 

Ans. **(3)**

Sol. (3) 

(A) (B) (C)

5. Thiamine & pyridoxine are also known respectively as:

(1) Vitamin B_2 and Vitamin E (2) Vitamin B_1 and Vitamin B_6

(3) Vitamin B_6 and Vitamin B_2 (4) Vitamin E & Vitamin B_2

Ans. **(2)**

Sol. NCERT

6. The set having ions which are coloured and paramagnetic both is:

(1) Cu^+, Zn^{2+}, Mn^{4+} (2) Sc^{3+}, V^{5+}, Ti^{4+} (3) Ni^{2+} , Mn^{7+}, Hg^{2+} (4) Cu^{2+}, Cr^{3+}, Sc^+

Ans. **(4)**

Sol.	**Ion**	**No. of unpaired e^-**
	Cu^{2+}	1
	Sc^{+}	2
	Cr^{3+}	3

This set is "paramagnetic & coloured"

7. Given below are the statements about diborane

(a) Diborane is prepared by the oxidation of $NaBH_4$ with I_2.

(b) Each boron atom is in sp^2 hybridized state.

(c) Diborane has one bridged 3 center-2 electron bond.

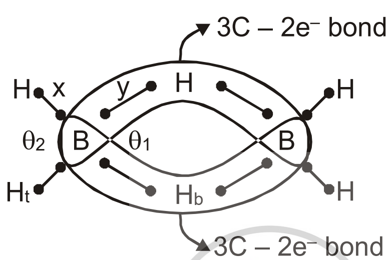
(d) Diborane is a planer molecule.

The option with correct statement(s) is:

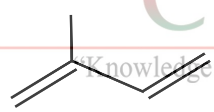
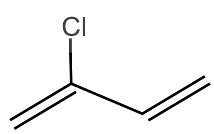
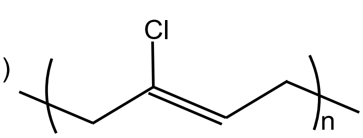
(1) (a) and (b) only (2) (a) only (3) (c) and (d) only (4) (c) only

Ans. **(2)**

Sol. $2NaBH_4 + I_2 \xrightarrow{\text{ether}} B_2H_6 + 2NaI + H_2\uparrow$



8. Match List-I with List-II :

List-I	**List-II**
(a) Chloroprene	(i) 
(b) Neoprene	(ii) 
(c) Acrylonitrile	(iii) 
(d) Isoprene	(iv) $CH_2=CH–CN$

Choose the correct answer from the options given below:

(1) (a) - (ii), (b) - (i), (c) - (iv), (d) - (iii)
(2) (a) - (ii), (b) - (iii), (c) - (iv), (d) - (i)

(3) (a) - (iii), (b) - (i), (c) - (iv), (d) - (ii)
(4) (a) - (iii), (b) - (iv), (c) - (ii), (d) - (i)

Ans. **(2)**

Sol. NCERT

9. Match List-I with List-II :

List-I	List-II
(a) Ba	(i) Organic solvent soluble compounds
(b) Ca	(ii) Outer electronic configuration $6s^2$
(c) Li	(iii) Oxalate insoluble in water
(d) Na	(iv) Formation of very strong monoacidic base

Choose the correct answer from the options given below:

(1) (a)–(iii), (b)–(ii), (c)-(iv) and (d)–(i)
(2) (a)–(ii), (b)–(iii), (c)-(i) and (d)–(iv)
(3) (a)–(i), (b)–(iv), (c)-(ii) and (d)–(iii)
(4) (a)–(iv), (b)–(i), (c)-(ii) and (d)–(iii)

Ans. (2)

Sol.

Ba	-	[Xe] $6s^2$
Ca	-	CaC_2O_4 is highly insoluble in water.
Li	-	Organic solvent soluble compounds due to covalent nature.
Na	-	Formation of very strong monoacidic base. eg. NaOH

10. Isotopes of hydrogen which emits low energy β^- particle with half life greater than 12 year is/are :

(1) Tritium
(2) Deuterium
(3) Deuterium and Tritium
(4) Protium

Ans. (1)

Sol. Only tritium is radioactive and emits low energy β particles ($t_{1/2}$, 12.33 years)

11. The water having more dissolved O_2 is:

(1) water at 4ºC
(2) polluted water
(3) boiling water
(4) water at 80ºC

Ans. (1)

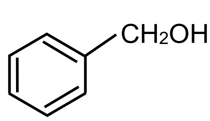
Sol. Solubility of oxygen is increase with decrease in temperature.

12. Which purification technique is used for high boiling organic liquid compound (decomposes near its boiling point)?

(1) Steam distillation
(2) Fractional distillation
(3) Simple distillation
(4) Reduced pressure distillation

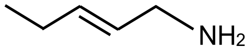
Ans. (4)

13. Which one of the following compounds does not exhibit resonance?

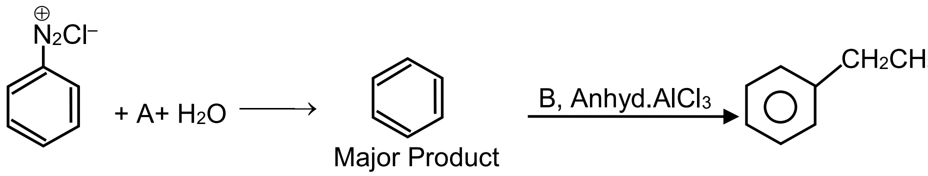
(1) $CH_3CH_2CH_2CONH_2$
(2) $CH_3CH_2OCH=CH_2$
(3) 

(4) $CH_3CH_2CH=CHCH_2NH_2$

Ans. **(4)**

Sol. 

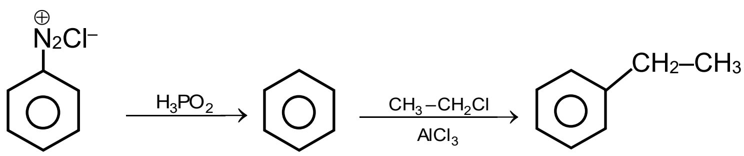
has no conjugation between π-bond and lone-pair hence there will be no resonance in this compound.

14. 

In the chemical reaction given above A & B respectively are:

(1) H_3PO_2 and CH_3CH_2Cl
(2) CH_3CH_2Cl and H_3PO_2
(3) H_3PO_2 and CH_3CH_2OH
(4) CH_3CH_2OH and H_3PO_2

Ans. **(1)**

Sol. 

15. Which one of the following statements for D.I. Mendeleeff, is incorrect?

(1) He authored the textbook-Principles of Chemistry.
(2) Element with atomic number 101 is named after him.
(3) At the time, he proposed Periodic Table of elements structure of atom was known.
(4) He invented accurate barometer.

Ans. **(3)**

Sol. NCERT based.

Preliminary work for his great textbook “Principles of Chemistry” led Mendeleev to propose the **Periodic Law** and to construct his **Periodic Table** of elements. At that time, the structure of atom was unknown and Mendeleev’s idea to consider that the properties of the elements were in someway related to their atomic masses was a very imaginative one.

You will notice from the modern Period Table (Fig.) that Mendeleev’s name has been immortalized by naming the element with atomic number 101, as Mendelevium.

16. Match List-I with List-II:

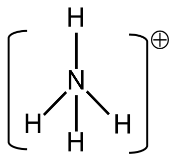
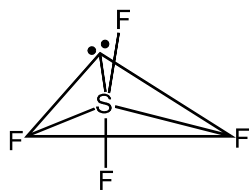
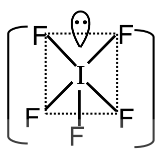
List-I (Species)	**List-II (Hybrid orbitals)**
(a) SF_4	(i) sp^3d^2
(b) IF_5	(ii) d^2sp^3
(c) $NO_2^{\oplus}$,	(iii) sp^3d
(d) $NH_4^{\oplus}$	(iv) sp^3
	(v) sp

Choose the correct answer from the options given below:

(1) (a)–(ii), (b)–(i), (c)-(iv) and (d)–(v)
(2) (a)–(iv), (b)–(iii), (c)-(ii) and (d)–(v)
(3) (a)–(i), (b)–(ii), (c)-(v) and (d)–(iii)
(4) (a)–(iii), (b)–(i), (c)-(v) and (d)–(iv)

Ans. **(4)**

Sol.

$NH_4^{\oplus}$		sp^3
$NO_2^{\oplus}$	$O = \overset{\oplus}{N} = O$	sp
SF_4		sp^3d
IF_5		sp^3d^2

17. Which one of the following group-15 hydride is the strongest reducing agent?

(1) BiH_3 (2) SbH_3 (3) AsH_3 (4) PH_3

Ans. **(1)**

Sol. NH_3

PH_3

AsH_3

SbH_3

BiH_3

As we move down the group reducing power is increase.

18. Sulphide ion is soft base and its ores are common for metals.

(a) Pb (b) Al (c) Ag (d) Mg

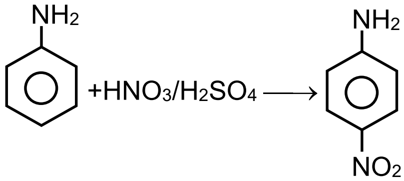
Choose the correct answer from the options given below:

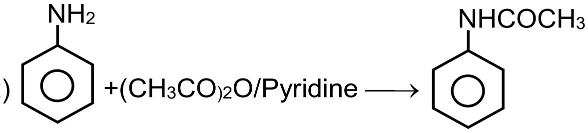
(1) (c) and (d) only (2) (a) and (d) only (3) (a) and (c) only (4) (a) and (b) only

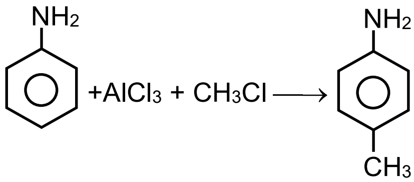
Ans. **(3)**

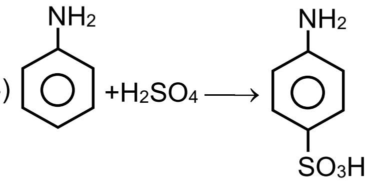
Sol. Sulphide ore ⇒ Pbs, Ag_2S, $CuFeS_2$, ZnS.

19. Which one of the following reactions does not occur?

(1) 

(2) 

(3) 

(4) 

Ans. **(3)**

Sol. Aniline does not give Friedel craft reaction.

20. Which one of the following 0.06 M aqueous solutions has lowest freezing point?

(1) $C_6H_{12}O_6$ (2) $Al_2(SO_4)_3$ (3) KI (4) K_2SO_4

Ans. **(2)**

Sol. $\Delta T_f = i\,K_f m$

Greater the i value lower will be freezing point

Section : Chemistry Section B

21. Value of K_P for the equilibrium reaction $N_2O_{4(g)} \rightleftharpoons 2NO_{2(g)}$ at 288 K is 47·9. The K_C for this reaction at same temperature is ___________. (Nearest integer)

(R = 0.083 L bar K^{-1} mol^{-1})

Ans. **(2)**

Sol. $K_P = K_C(RT)^{\Delta n_g}$

$47{\cdot}9 = K_C\,(0{\cdot}083 \times 288)^1$

$K_C = 2$

22. Methylation of 10 g of benzene gave 9.2 g of toluene. Calculate the percentage yield of toluene ___________. (Nearest integer)

Ans. **(78)**

Sol. $\underset{\frac{10}{78}\text{mole}}{C_6H_6} \xrightarrow{\text{Methylatio n}} \underset{\frac{10}{78}\text{mole}}{C_6H_5CH_3}$

$(W_{\text{theoritical}}) = \frac{10}{78} \times 92$

$$\% \text{ yield} = \frac{W_{actual}}{W_{theoritica\,l}} \times 100$$

$$= \left[\frac{9.2}{10 \times 92} \times 78\right] \times 100 = 78\%$$

23. Assume a cell with the following reaction

$Cu_{(s)} + 2Ag^{+}\ (1 \times 10^{-3}M) \rightarrow Cu^{2+}\ (0.250\ M) + 2\ Ag_{(s)}$

$E^{0}_{cell} = 2.97\ V$

E_{cell} for the above reaction is ___________ V. (Nearest integer)

[Given : log 2.5 = 0.3979, T = 298 K]

Ans. **(3)**

Sol. $E_{cell} = E°_{cell} - \frac{0.059}{2}\log\frac{[Cu^{2+}]}{[Ag^{+}]^2} = 2.97 - \frac{0.059}{2}\log\left\{\frac{0.250}{(10^{-3})^2}\right\}$

$= 2.97 - 0.177\ (-0.602) = 3.07$

Ans. = 3

24. If the standard molar enthalpy change for combustion of graphite powder is -2.48×10^{2} kJ mol^{-1}, the amount of heat generated on combustion of 1 g of graphite powder is __________ kJ. (Nearest integer)

Ans. **(21)**

Sol. C(graphite) + $O_2 \rightarrow CO_2(g)$ $\Delta H = -2.48$ kJ mole

1 gram

Total heat released = $2.48 \times \frac{1}{12} \times 10^{2}$

= 20.67

Ans. 21

25. The total number of unpaired electrons present in $[Co(NH_3)_6]Cl_2$ and $[Co(NH_3)_6]Cl_3$ is __________.

Ans. **(1)**

Sol. Complex

(i) $[Co(NH_3)_6]\ Cl_2 \Rightarrow Co^{2+} = 3d^7$ unpaired electron = 1

(ii) $[Co(NH_3)_6]\ Cl_3 \Rightarrow Co^{3+} = 3d^6$ unpaired electron = 0

Total unpaired electrons = 1

26. If the concentration of glucose ($C_6H_{12}O_6$) in blood is 0.72 g L^{-1}, the molarity of glucose in blood is _________ × 10^{-3} M. (Nearest integer)

(Given : Atomic mas of C = 12, H = 1, O = 16 u)

Ans. **(4)**

Sol. $M = \dfrac{W_{solute}}{M_{solute} \times V_{soln}(\text{inlit})}$ **=** $\dfrac{0.72}{180}$

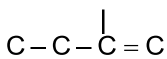
$= 0.004 = 4 \times 10^{-3}$

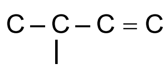
27. The number of acyclic structural isomers (including geometrical isomers) for pentene are __________.

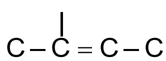
Ans. **(6)**

Sol. C–C–C–C=C (1)

C–C–C=C–C (2)

 (1)

 (1)

 (1)

28. A copper complex crystallising in a CCP lattice with a cell edge of 0.4518 nm has been revealed by employing X-ray diffraction studies. The density of a copper complex is found to be 7.62 g cm^{-3}. The molar mass of copper complex is _________ g mol^{-1}. (Nearest integer)

[Given : $N_A = 6.022 \times 10^{23}$ mol^{-1}]

Ans. **(106)**

Sol. $d = \left\{\dfrac{Z \times M}{N_A \times \text{Volume}}\right\}$

$7.62 = \dfrac{4 \times M}{6.022 \times 10^{23} \times [0.4518 \times 10^{-7}]^3}$

$M = \dfrac{7.62 \times 6.022 \times 10^{23} \times [0.4518 \times 10^{-7}]^3}{4}$

= 1.057 × p10^2

= 105.7 gram/mole

Ans. 106

29. $N_2O_{5(g)} \rightarrow 2NO_{2(g)} + \frac{1}{2}O_{2(g)}$

In the above first order reaction the initial concentration of N_2O_5 is 2.40×10^{-2} mol L^{-1} at 318 K. the concentration of N_2O_5 after 1 hour was 1.60×10^{-2} mol L^{-1}. The rate constant of the reaction at 318 K is ________ $\times 10^{-3}$ min^{-1}. (Nearest integer)

[Given : log 3 = 0.477, log 5 = 0.699]

Ans. **(7)**

Sol.

	$2N_2O_5(g)$	$\rightarrow$	$2NO_2(g)$	+	$\frac{1}{2}O_2(g)$
Initial	$2{\cdot}4 \times 10^{-2}$ M		–		–
After 1 hour	$1{\cdot}6 \times 10^{-2}$ M				

$$K = \frac{1}{t}\ln\left(\frac{a}{a-x}\right)$$

$$k = \frac{2{\cdot}303}{60}\log\left(\frac{2{\cdot}4\times10^{-2}}{1{\cdot}6\times10^{-2}}\right)$$

$$k = \frac{2{\cdot}303}{60}\log\left(\frac{3}{2}\right)$$

$k = 0{\cdot}0069 = 6.9 \times 10^{-3}\ min^{-1}$

$k = 7$

30. Number of electrons that Vanadium (Z = 23) has in p-orbitals is equal to ________.

Ans. **(12)**

Sol. ${}_{23}V = 1s^2\, 2s^2\, 2p^6\, 3s^2\, 3p^6\, 3d^3\, 4s^2$

MATHEMATICS

1. The values of λ and μ such that the system of equations
$x + y + z = 6, 3x + 5y + 5z = 26, x + 2y + \lambda z = \mu$ has no solution, are:
λ तथा μ के वे मान जिनके लिए समीकरण निकाय
$x + y + z = 6, 3x + 5y + 5z = 26, x + 2y + \lambda z = \mu$ का कोई हल नहीं है:
(1) $\lambda = 3, \mu \neq 10$ (2) $\lambda \neq 2, \mu = 10$ (3) $\lambda = 2, \mu \neq 10$ (4) $\lambda = 3, \mu = 5$

Ans. **(3)**

Sol. For no solution $\Delta = 0$

$\Delta = 0$

$$\begin{vmatrix} 1 & 1 & 1 \\ 3 & 5 & 5 \\ 1 & 2 & \lambda \end{vmatrix} = 0$$

$\Rightarrow 1(5\lambda - 10) - 1(3\lambda - 5) + 1(6 - 5) = 0$

$\Rightarrow 2\lambda - 4 = 0$

$\Rightarrow \lambda = 2$

$$\Delta_1 = \begin{vmatrix} 6 & 1 & 1 \\ 26 & 5 & 5 \\ \mu & 2 & 2 \end{vmatrix} = 0$$

$$\Delta_2 = \begin{vmatrix} 1 & 6 & 1 \\ 3 & 26 & 5 \\ 1 & \mu & 2 \end{vmatrix} = 1(52 - 5\mu) - 6(6 - 5) + 1(3\mu - 26)$$

$= 52 - 5\mu - 6 + 3\mu - 26$

$\Delta_2 = 20 - 2\mu$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 6 \\ 3 & 5 & 26 \\ 1 & 2 & \mu \end{vmatrix} = 1(5\mu - 52) - 1(3\mu - 26) + 6(6 - 5)$$

$\Delta_3 = 2\mu - 20$

Case-I

$\lambda = 2, \mu = 10 \Rightarrow \Delta = 0, \Delta_1 = 0, \Delta_2 = 0, \Delta_3 = 0$

system of equations are

$x + y + z = 6$

$3x + 5y + 5z = 26$

$x + 2y + 2z = 10$ has infinite many solutions

Case - II

$\lambda = 2, \mu \neq 10 \Rightarrow \Delta = 0, \Delta_1 = 0, \Delta_2 \neq 0, \Delta_3 \neq 0$

system has no solution

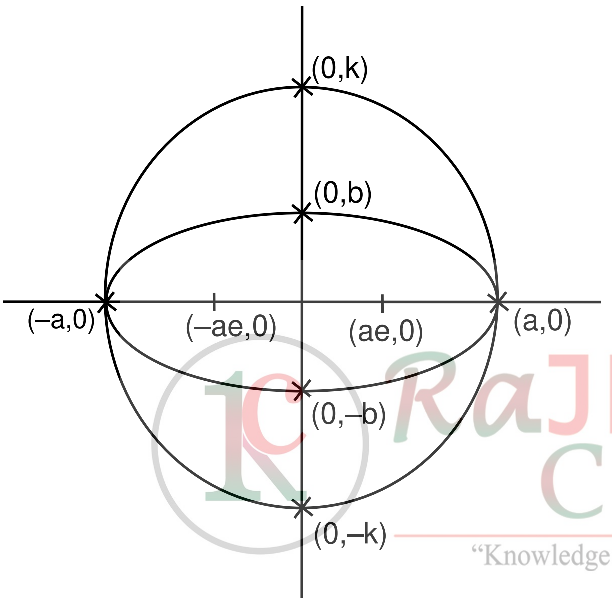
2. Let $E_1 : \frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, a > b. Let E_2 be another ellipse such that it touches the end points of major axis of E_1 and the foci of E_2 are the end points of minor axis of E_1. If E_1 and E_2 have same eccentricities, then its value is:

माना $E_1 : \frac{x^2}{a^2}+\frac{y^2}{b^2}=1, a > b$ एक दीर्घवृत्त है। माना E_2 एक और दीर्घवृत्त है, जो E_1 के दीर्घ अक्ष के छोरों को स्पर्श करता है तथा E_2 की नाभियाँ, E_1 के लघु अक्ष के छोरों पर है। यदि E_1 तथा E_2 की उत्केंन्द्रता बराबर है, तो उसका मान है:

(1) $\frac{-1+\sqrt{6}}{2}$ (2) $\frac{-1+\sqrt{3}}{2}$ (3) $\frac{-1+\sqrt{5}}{2}$ (4) $\frac{-1+\sqrt{8}}{2}$

Ans. **(3)**

Sol.



Eccentricity of E_1 is e $\Rightarrow e^2 = 1-\frac{b^2}{a^2}$

Eccentricity of E_2 is e $\Rightarrow e^2 = 1-\frac{a^2}{k^2}$

So, $e^2 = 1-\frac{b^2}{a^2} = 1-\frac{a^2}{k^2} \Rightarrow k = \frac{a^2}{b}$ ....(i)

Also ke = b ....(ii)

From equation (i) and (ii) $e = \frac{b^2}{a^2}$

Since $e^2 = 1-\frac{b^2}{a^2} \Rightarrow e^2 = 1-e$

$\Rightarrow e^2 + e - 1 = 0$

$\Rightarrow e = \frac{\sqrt{5}-1}{2}$

3. Let S_n denote the sum of first n- terms of an arithmetic progression. If $S_{10} = 530$, $S_5 = 140$, then $S_{20} - S_6$ is equal to:

माना एक समान्तर श्रेढ़ी के प्रथम n पदों का योग S_n है। यदि $S_{10} = 530$, तथा $S_5 = 140$ है, तो $S_{20} - S_6$ बराबर है:

(1) 1872 (2) 1842 (3) 1862 (4) 1852

Ans. **(3)**

Sol. $S_{10} = 530$

$$\frac{10}{2}[2a+9d] = 530$$

$\Rightarrow 2a + 9d = 106$ … (1)

$S_5 = 140$

$$\Rightarrow \frac{5}{2}[2a+4d] = 140$$

$\Rightarrow 2a + 4d = 56$ … (2)

$\Rightarrow 5d = 50$

$d = 10$

$a = 8$

Now,

$\Rightarrow S_{20} - S_6 =$

$\Rightarrow 10[2a + 19d] - 3[2a + 5d]$

$\Rightarrow 14a + 175d$

$14 \times 8 + (175)10 = 1862$

4. Let $f : R \to R$ be defined as $f(x) = \begin{cases} \frac{x^3}{(1-\cos 2x)^2}\log_e\left(\frac{1+2xe^{-2x}}{(1-xe^{-x})^2}\right); & x \neq 0 \\ \alpha ; & x = 0 \end{cases}$

If f is continuous at x = 0, then α is equal to:

माना $f : R \to R$, $f(x) = \begin{cases} \frac{x^3}{(1-\cos 2x)^2}\log_e\left(\frac{1+2xe^{-2x}}{(1-xe^{-x})^2}\right); & x \neq 0 \\ \alpha ; & x = 0 \end{cases}$

द्वारा परिभाषित है। यदि $x = 0$ पर f संतत है, तो α बराबर है:

(1) 2 (2) 3 (3) 1 (4) 0

Ans. **(3)**

Sol. $$\lim_{x\to 0}\frac{x^3}{4\sin^4 x}\log_e\left(\frac{1+2xe^{-2x}}{(1-xe^{-x})^2}\right)$$

$$\lim_{x\to 0}\frac{1}{4\left(\frac{\sin x}{x}\right)^4 x}\log_e\left(\frac{1+2xe^{-2x}}{(1-xe^{-x})^2}\right)$$

$$\lim_{x\to 0}\frac{1}{4x}\log_e\left(\frac{1+2xe^{-2x}}{(1-xe^{-x})^2}\right)\times\frac{x^4}{\sin^4 x}$$

$$\lim_{x\to 0}\frac{1}{4}\left[\frac{2e^{-2x}\log(1+2xe^{-2x})}{2xe^{-2x}}+\frac{2e^{-x}\log(1-xe^{-x})}{-xe^{-x}}\right]\times\frac{x^4}{\sin^4 x}$$

$$\lim_{x\to 0}\frac{1}{4}\left[2e^{-2x}+2e^{-x}\right]\times\frac{x^4}{\sin^4 x}=\frac{2+2}{4}=1$$

If f is continuous at x = 0 then

$f(0) = \lim_{x\to 0} f(x)$

$\alpha = 1$

5. If the shortest distance between the straight lines $3(x-1) = 6(y-2) = 2(z-1)$ and $4(x-2) = 2(y-\lambda) = (z-3)$, $\lambda \in R$ is $\frac{1}{\sqrt{38}}$, then the integral value of λ is equal to :

यदि सरल रेखाओं $3(x-1) = 6(y-2) = 2(z-1)$ तथा $4(x-2) = 2(y-\lambda) = (z-3)$, $\lambda \in R$ के बीच की न्यूनतम दूरी $\frac{1}{\sqrt{38}}$ है, तो λ का पूर्णांक मान बराबर है:

(1) 2 (2) 5 (3) 3 (4) –1

Ans. **(3)**

Sol. Lines are $\frac{x-1}{\frac{1}{3}}=\frac{y-2}{\frac{1}{6}}=\frac{z-1}{\frac{1}{2}}$ and $\frac{(x-2)}{\frac{1}{4}}=\frac{(y-\lambda)}{\frac{1}{2}}=\frac{(z-3)}{1}$

Shortest distance $=\frac{1}{\sqrt{38}}$

$\vec{a}_2=2\hat{i}+\lambda\hat{j}+3\hat{k},\ \vec{a}_1=\hat{i}+2\hat{j}+\hat{k}$

$\vec{b}_1=\frac{1}{3}\hat{i}+\frac{1}{6}\hat{j}+\frac{1}{2}\hat{k},\ \vec{b}_2=\frac{1}{4}\hat{i}+\frac{1}{2}\hat{j}+\hat{k}$

$\vec{b}_1\times\vec{b}_2=-\frac{1}{12}\hat{i}-\frac{5}{24}\hat{j}+\frac{1}{8}\hat{k}$

$|\vec{b}_1\times\vec{b}_2|=\frac{\sqrt{38}}{24}$

$$\frac{1}{\sqrt{38}}=\left|\frac{(\vec{a}_2-\vec{a}_1).(\vec{b}_1\times\vec{b}_2)}{|\vec{b}_1\times\vec{b}_2|}\right|$$

$$\Rightarrow \frac{1}{\sqrt{38}}=\frac{\left|(\hat{i}+(\lambda-2)\hat{j}+2\hat{k}).\left(-\frac{1}{12}\hat{i}-\frac{5}{24}\hat{j}+\frac{1}{8}\hat{k}\right)\right|}{\frac{\sqrt{38}}{24}}$$

$$\Rightarrow \frac{1}{24}=\left|-\frac{1}{12}+(\lambda-2)\left(\frac{-5}{24}\right)+\frac{1}{4}\right|$$

$\Rightarrow 1 = |14-5\lambda|$

$\Rightarrow \lambda=\frac{13}{5}$ (rejected)

$\Rightarrow \lambda = 3$

6. Let f : R → R be defined as

$$f(x) = \begin{cases} -\frac{4}{3}x^3 + 2x^2 + 3x, & x > 0 \\ 3xe^x, & x \le 0 \end{cases}$$

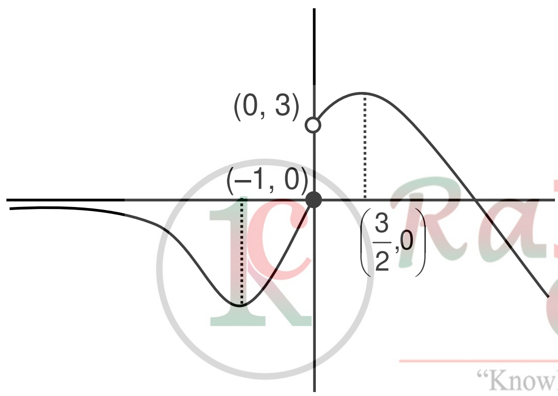
Then f is increasing function in the interval.

माना f : R → R, $f(x) = \begin{cases} -\frac{4}{3}x^3 + 2x^2 + 3x, & x > 0 \\ 3xe^x, & x \le 0 \end{cases}$ द्वारा परिभाषित है। तो निम्न में से किस अन्तराल में फलन f वर्धमान है ?

(1) $\left(-\frac{1}{2}, 2\right)$ (2) (–3, –1) (3) (0,2) (4) $\left(-1, \frac{3}{2}\right)$

Ans. **(4)**

Sol. $f'(x) = \begin{cases} -4x^2 + 4x + 3 & , x > 0 \\ 3(x.e^x + e^x) & , x \le 0 \end{cases}$



7. Let a line L : 2x + y = k, k > 0 be a tangent to the hyperbola $x^2 - y^2 = 3$. If L is also a tangent to the parabola $y^2 = \alpha x$, then α is equal to:

माना एक रेखा L : 2x + y = k, k > 0 , अतिपरवलय $x^2 - y^2 = 3$ को स्पर्श करती है। यदि रेखा L परवलय, $y^2 = \alpha x$, को भी स्पर्श करती है, तो α बराबर है:

(1) 24 (2) –24 (3) 12 (4) –12

Ans. **(2)**

Sol. Given slope of line (m) = – 2

slope form of tangent to the curve $x^2 - y^2 = 3$ is $y = mx \pm \sqrt{a^2m^2 - b^2}$

$\Rightarrow y = -2x \pm 3$

On comparing, with the equation 2x + y = k, (k > 0) ⇒ k = 3

Now, slope form of tangent to the parabola $y^2 = \alpha x$ is $y = mx + \frac{\alpha}{4m}$

But m = – 2 so

$$y = -2x + \frac{\alpha}{4(-2)}$$

$$\Rightarrow 3 = \frac{\alpha}{4 \times (-2)}$$

$\alpha = -24$

8. Four dice are thrown simultaneously and the numbers shown on these dice are recorded in 2 × 2 matrices. The probability that such formed matrices have all different entries and are non-singular, is:
चार पासे एक साथ फेंके जाते है और उन पर आई संख्याओं से 2 × 2 आव्यूह बनाए जाते है। ऐसे बने आव्यूहों, जिनकी सभी प्रविष्टियाँ विभिन्न है तथा जो व्युत्क्रमणीय भी हैं, की प्रायिकता है:

(1) $\frac{22}{81}$ (2) $\frac{23}{81}$ (3) $\frac{45}{162}$ (4) $\frac{43}{162}$

Ans. **(4)**

Sol. Number of matrices having distinct elements = $^6C_4 \times 4!$

$\Rightarrow$ Number of non singular matrices having distinct elements

= $^6C_4 \times 4!$ – Number of singular matrices having distinct elements

$$X = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$|x| = ad - bc = 0$

$$\left.\begin{matrix} (1,6) & (3,2) \\ (3,4) & (6,2) \end{matrix}\right\} 8+8 \text{ possibilities}$$

$\Rightarrow$ Number of non singular matrices having distinct elements

= $^6C_4 \times 4! - 16 = 344$

So required probability = $\frac{344}{6^4} = \frac{43}{162}$

9. Let three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ be such that $\vec{a} \times \vec{b} = \vec{c}$, $\vec{b} \times \vec{c} = \vec{a}$ and $|\vec{a}| = 2$. Then which one of the following is not true?

(1) Projection of $\vec{a}$ on $(\vec{b} \times \vec{c})$ is 2

(2) $\left|3\vec{a} + \vec{b} - 2\vec{c}\right|^2 = 51$

(3) $[\vec{a}\ \vec{b}\ \vec{c}] + [\vec{c}\ \vec{a}\ \vec{b}] = 8$

(4) $\vec{a} \times ((\vec{b} + \vec{c}) \times (\vec{b} - \vec{c})) = \vec{0}$

माना तीन सदिश $\vec{a}, \vec{b}$ तथा $\vec{c}$, $\vec{a} \times \vec{b} = \vec{c}$, $\vec{b} \times \vec{c} = \vec{a}$ तथा $|\vec{a}| = 2$ को संतुष्ट करते है। तो निम्न में से कौन सा कथन असत्य है?

(1) $(\vec{b} \times \vec{c})$ पर $\vec{a}$ का प्रक्षेप 2 है

(2) $\left|3\vec{a} + \vec{b} - 2\vec{c}\right|^2 = 51$

(3) $[\vec{a}\ \vec{b}\ \vec{c}] + [\vec{c}\ \vec{a}\ \vec{b}] = 8$

(4) $\vec{a} \times ((\vec{b} + \vec{c}) \times (\vec{b} - \vec{c})) = \vec{0}$

Ans. **(2)**

Sol. $|\vec{a}| = 2$

$\vec{a} \times \vec{b} = \vec{c} \Rightarrow [\vec{a}\vec{b}\vec{c}] = |\vec{c}|^2$

$\vec{b} \times \vec{c} = \vec{a} \Rightarrow [\vec{a}\vec{b}\vec{c}] = |\vec{a}|^2$

Hence $|\vec{a}|^2 = |\vec{c}|^2 = 4$

Also $\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{a}.\vec{c} = 0$

1. Projection of $\vec{a}$ on $\vec{b} \times \vec{c}$ = $\frac{\vec{a}.(\vec{b} \times \vec{c})}{|\vec{b} \times \vec{c}|} = \frac{\vec{a}.(\vec{b} \times \vec{c})}{|\vec{a}|} = \frac{(\vec{a} \times \vec{b}).\vec{c}}{|\vec{a}|} = \frac{|\vec{c}|^2}{|\vec{a}|} = |\vec{a}| = 2$ (correct)

2. $|3\vec{a}+\vec{b}-2\vec{c}|^2 = 9|\vec{a}|^2+|\vec{b}|^2+4|\vec{c}|^2+6(\vec{a}.\vec{b})-12(\vec{a}.\vec{c})-4(\vec{b}.\vec{c})$

$= 36 + |\vec{b}|^2 + 16 + 0+0+0$

$52 + |\vec{b}|^2 \neq 51$ (Incorrect)

3. $[\vec{a}\vec{b}\vec{c}]+[\vec{c}\vec{a}\vec{b}] = |\vec{a}|^2+|\vec{a}|^2 = 2|\vec{a}|^2 = 8$

4. $\vec{a}\times((\vec{b}+\vec{c})\times(\vec{b}\times\vec{c})) = \vec{a}\times(-2(\vec{b}\times\vec{c})) = -2\vec{a}\times(\vec{a}) = 0$ (correct)

10. Which of the following Boolean expressions is not a tautology?
निम्न में से कौन–सा बूलीय व्यंजक पुनरावृत्ति **नहीं** है?

(1) $(\sim p \Rightarrow q) \vee (\sim q \Rightarrow p)$
(2) $(p \Rightarrow \sim q) \vee (\sim q \Rightarrow p)$
(3) $(q \Rightarrow p) \vee (\sim q \Rightarrow p)$
(4) $(p \Rightarrow q) \vee (\sim q \Rightarrow p)$

Ans. **(1)**

Sol. $(\sim p \Rightarrow q)\vee(\sim q \Rightarrow p)$

$\Rightarrow (p\vee q)\vee(q\vee p)$

$\Rightarrow p\vee q$

(2) $(p \Rightarrow \sim q)\vee(\sim q \Rightarrow p)$

$\Rightarrow (\sim p\vee \sim q)\vee(q\vee p)$

$\Rightarrow t$

(3) $(q \Rightarrow p)\vee(\sim q \Rightarrow p)$

$\Rightarrow (\sim q\vee p)\vee(q\vee p)$

$\Rightarrow t$

(4) $(p \Rightarrow q)\vee(\sim q \Rightarrow p)$

$\Rightarrow (\sim p\vee q)\vee(q\vee p)$

$\Rightarrow t$

11. Let n denote the number of solutions of the equation $z^2 + 3\bar{z} = 0$, where z is a complex number. Then value of $\sum_{k=0}^{\infty} \frac{1}{n^k}$ is equal to :

माना सम्मिश्र संख्या z के लिए समीकरण $z^2 + 3\bar{z} = 0$ के मूलों की संख्या n है। तो $\sum_{k=0}^{\infty} \frac{1}{n^k}$ का मान बराबर है:

(1) $\frac{3}{2}$
(2) 2
(3) 1
(4) $\frac{4}{3}$

Ans. **(4)**

Sol. Let z = x + iy

$(x + iy)^2 + 3(x - iy) = 0$

$x^2 - y^2 + 2ixy + 3x - 3iy = 0$

$x^2 - y^2 + 3x = 0$ & $2xy - 3y = 0$

Case-1: $y = 0$

$x^2 - y^2 + 3x = 0$

$\Rightarrow x = 0$ or $x = -3$

Solutions are $z=0$ and $z=-3$

Case-2: $x = \dfrac{3}{2}$

$x^2 - y^2 + 3x = 0$

$\Rightarrow\ y = \dfrac{3\sqrt{3}}{2}$ or $y = \dfrac{-3\sqrt{3}}{2}$

Solutions are $z = \dfrac{3}{2} + i\dfrac{3\sqrt{3}}{2}$ and $z = \dfrac{3}{2} - i\dfrac{3\sqrt{3}}{2}$

Total number of solutions = n = 4

So $\displaystyle\sum_{k=0}^{\infty} \frac{1}{4^k} = \frac{1}{1-\frac{1}{4}} = \frac{4}{3}$ Ans.

12. Let $A = [a_{ij}]$ be a real matrix of order 3×3, such that $a_{i1} + a_{i2} + a_{i3} = 1$, for 1, 2, 3. Then, the sum of all the entries of the matrix A^3 is equal to:

माना $A = [a_{ij}]$ कोटि 3×3 का एक वास्तविक आव्यूह इस प्रकार है कि प्रत्येक $i = 1,2,3$ के लिए $a_{i1} + a_{i2} + a_{i3} = 1$ है। तो आव्यूह A^3 की सभी प्रविष्टियों का योग बराबर है:

(1) 9 (2) 1 (3) 2 (4) 3

Ans. **(4)**

Sol. $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

$a + b + c = 1$

$d + e + f = 1$

$g + h + i = 1$

Let suppose a matrix $Y = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

So,

$$AY = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} a+b+c \\ d+e+f \\ g+h+i \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$AY = Y$ ……..(1)

Substitute $Y = AY$ in equation (1)

So, $A^2Y = AY = Y$

Again substitute $Y = AY$

$\Rightarrow A^3Y = A^2Y = AY = Y$

So, $A^3Y = Y$

Let us suppose $A^3 = \begin{bmatrix} A & B & C \\ D & E & F \\ G & H & I \end{bmatrix}$

$$\begin{bmatrix} A & B & C \\ D & E & F \\ G & H & I \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} A+B+C \\ D+E+F \\ G+H+I \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$A + B + C = 1$
$D + E + F = 1$
$G + H + I = 1$
So, $A + B + C + D + E + F + G + H + I = 3$
Sum of elements of $A^3 = 3$

13. Let L be the line of intersection of planes $\vec{r}\cdot(\hat{i}-\hat{j}+2\hat{k})=2$ and $\vec{r}\cdot(2\hat{i}+\hat{j}-\hat{k})=2$. If P (α, β, γ) is the foot of perpendicular on L from the point (1, 2, 0) then the value of $35(\alpha + \beta + \gamma)$ is equal to:
माना L, समतलों $\vec{r}\cdot(\hat{i}-\hat{j}+2\hat{k})=2$ तथा $\vec{r}\cdot(\hat{i}-\hat{j}+2\hat{k})=2$ की प्रतिच्छेदन रेखा है। यदि बिन्दु (1, 2, 0) से रेखा L पर डाले गए लम्ब का पाद P (α, β, γ) है, तो $35(\alpha + \beta + \gamma)$ का मान बराबर है:

(1) 119 (2) 134 (3) 101 (4) 143

Ans. **(1)**

Sol. Given planes are
$x - y + 2z = 2$ and $2x + y - z = 2$
$z = 0$
$\Rightarrow x - y = 2$ and $2x + y = 2$

(1) and (2), $3x = 4$ $x = \frac{4}{3} \Rightarrow y = -\frac{2}{3}$

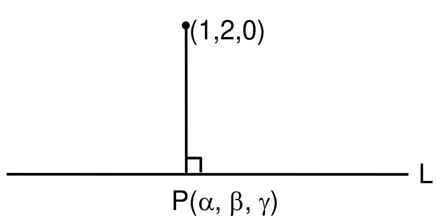
$\therefore \therefore \left(\frac{4}{3}, \frac{-2}{3}, 0\right)$ lies on line of intersection of planes

for dr's of line $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ 2 & 1 & -1 \end{vmatrix}$

$= \hat{i}(1-2) - \hat{j}(-1-4) + \hat{k}(1+2)$
$= -\hat{i} + 5\hat{j} + 3\hat{k}$

$\therefore$ line of intersection is

$$\frac{x-\frac{4}{3}}{-1} = \frac{y+\frac{2}{3}}{5} = \frac{z-0}{3} = \lambda$$



$x = \frac{4}{3} - \lambda$ and $z = 3\lambda$

$y = 5\lambda - \frac{2}{3}$

$$\left(\frac{4}{3} - \lambda - 1\right)(-1) + \left(5\lambda - \frac{2}{3} - 2\right)(5) + (3\lambda)(3) = 0$$

$$\left(\frac{1}{3}-\lambda\right)(-1)+\left(5\lambda-\frac{8}{3}\right)(5)+9\lambda=0$$

$$\Rightarrow \lambda-\frac{1}{3}+25\lambda-\frac{40}{3}+9\lambda=0$$

$$\Rightarrow 35\lambda=\frac{41}{3}\Rightarrow \lambda=\frac{41}{105}$$

$$\text{so, } \alpha=\frac{4}{3}-\frac{41}{105}=\frac{99}{105}$$

$$\beta=5\left(\frac{41}{105}\right)-\frac{2}{3}=\frac{205-70}{105}=\frac{135}{105}$$

$$\gamma=\frac{123}{105}$$

$$\Rightarrow 35(\alpha+\beta+\gamma)=\frac{(99+135+123)}{105}\times 35=119$$

14. If $\int_0^{100\pi}\frac{\sin^2 x}{e^{\left(\frac{x}{\pi}-\left[\frac{x}{\pi}\right]\right)}}dx=\frac{\alpha\pi^3}{1+4\pi^2}$, $\alpha\in R$, where [x] is the greatest less than or equal to x, then the value of α is:

यदि $\int_0^{100\pi}\frac{\sin^2 x}{e^{\left(\frac{x}{\pi}-\left[\frac{x}{\pi}\right]\right)}}dx=\frac{a\pi^3}{1+4\pi^2}$, $\alpha\in R$ है, जबकि [x], महत्तम पूर्णांक $\le x$ है, तो α बराबर है:

(1) $50(e-1)$ (2) $200(1-e^{-1})$ (3) $150(e^{-1}-1)$ (4) $100(1-e)$

Ans. **(2)**

Sol. $$\int_0^{100\pi}\frac{\sin^2 x}{e^{\left\{\frac{x}{\pi}\right\}}}dx$$

$$\Rightarrow 100\int_0^{\pi}\frac{\sin^2 x}{e^{x/\pi}}dx$$

$$\Rightarrow 50\int_0^{\pi}e^{-x/\pi}[1-\cos 2x]dx$$

$$\Rightarrow 50\left[e^{-x/\pi}\times(-\pi)\right]_0^{\pi}-50\int_0^{\pi}e^{-x/\pi}\cos 2x\,dx$$

$$\Rightarrow 50\times(-\pi)(e^{-1}-1)-\frac{50\times\left[e^{-x/\pi}\left(\frac{-1}{\pi}\times\cos 2x+2\sin 2x\right)\right]_0^{\pi}}{\left(\frac{1}{\pi^2}+4\right)}$$

$$\Rightarrow -50\pi(e^{-1}-1)-\frac{50\pi^2}{(1+4\pi^2)}\left[e^{-1}\left(\frac{-1}{\pi}\right)+\frac{1}{\pi}\right]$$

$$\Rightarrow \frac{200\pi^3(1-e^{-1})}{1+4\pi^2}$$ So $\alpha=200(1-e^{-1})$

15. The number of solution of $\sin^7 x + \cos^7 x = 1$, $x \in [0, 4\pi]$ is equal to :
समीकरण $\sin^7 x + \cos^7 x = 1$, के $x \in [0, 4\pi]$ में हलों की संख्या है:
(1) 11 (2) 9 (3) 5 (4) 7

Ans. **(3)**

Sol. $\sin^2 x + \cos^2 x = 1$, $\sin^2 x \le 1$ and $\cos^2 x \le 1$

$\sin^7 x \le \sin^2 x$

$\cos^7 x \le \cos^2 x$

so, $\sin^7 x + \cos^7 x \le 1$

$\sin^7 x + \cos^7 x = 1$ when $\sin^7 x = \sin^2 x$ & $\cos^7 x = \cos^2 x$

Case-1 : $\sin x = 0, \cos x = 1 \Rightarrow x = 0, 2\pi, 4\pi$

Case-2 : $\sin x = 1, \cos x = 0 \Rightarrow x = \frac{\pi}{2}, \frac{5\pi}{2}$

Total number of solutions = 5

16. Let the circle S: $36x^2 + 36y^2 - 108x + 120y + C = 0$ be such that it neither intersects nor touches the co-ordinate axes. If the point of intersection of the lines, $x - 2y = 4$ and $2x - y = 5$ lies inside the circle S, then:
माना: वृत्त S: $36x^2 + 36y^2 - 108x + 120y + C = 0$ न तो निर्देशांक अक्षों को काटता है और न ही उनको स्पर्श करता है। यदि रेखाओं, $x - 2y = 4$ तथा $2x - y = 5$ का प्रतिच्छेदन बिन्दु, वृत्त S के अन्दर स्थित है, तो
(1) $\frac{25}{9} < C < \frac{13}{3}$ (2) $100 < C < 165$ (3) $100 < C < 156$ (4) $81 < C < 156$

Ans. **(3)**

Sol. Intersection point of $2x - y = 5$ and $x - 2y = 4$ is $(2, -1)$

So, $(2, -1)$ lies indie the circle $\Rightarrow S_1 < 0$

$36(2)^2 + 36(-1)^2 - 108(2) + 120(-1) + c < 0$

$c < 156$ ....(i)

$\because$ circle $36x^2 + 36y^2 - 108x + 120y + c = 0$ neither touches nor cuts the co-ordinate axis so

$$g^2 - c < 0 \Rightarrow \left(\frac{-3}{2}\right)^2 - \frac{c}{36} < 0 \Rightarrow c > 81 \text{..........(ii)}$$

$$\text{and } f^2 - c < 0 \Rightarrow \left(\frac{5}{3}\right)^2 - \frac{c}{36} < 0 \Rightarrow c > 100 \text{ ....(iii)}$$

From (i), (ii) and (iii)

$100 < c < 156$

17. Let a vector $\vec{a}$ be coplanar with vectors $\vec{b} = 2\hat{i} + \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} + \hat{k}$. If $\vec{a}$ is perpendicular to $\vec{d} = 3\hat{i} + 2\hat{j} + 6\hat{k}$, and $|\vec{a}| = \sqrt{10}$. Then a possible value of $[\vec{a}\ \vec{b}\ \vec{c}] + [\vec{a}\ \vec{b}\ \vec{d}] + [\vec{a}\ \vec{c}\ \vec{d}]$ is equal to :
माना एक सदिश $\vec{a}$ सदिशों $\vec{b} = 2\hat{i} + \hat{j} + \hat{k}$ तथा $\vec{c} = \hat{i} - \hat{j} + \hat{k}$ के सहतलीय है। यदि $\vec{a}$,सदिश $\vec{d} = 3\hat{i} + 2\hat{j} + 6\hat{k}$ पर लम्बवत है और $|\vec{a}| = \sqrt{10}$ है, तो $[\vec{a}\ \vec{b}\ \vec{c}] + [\vec{a}\ \vec{b}\ \vec{d}] + [\vec{a}\ \vec{c}\ \vec{d}]$ का एक संभावित मान है:
(1) – 42 (2) – 38 (3) – 40 (4) – 29

Ans. **(1)**

Sol. Let $\vec{a}=\vec{b}+\lambda\vec{c}$. Where $\vec{a},\vec{b},\vec{c}$ are coplanar.

Now $\vec{a}\perp\vec{d} \Rightarrow \vec{a}\cdot\vec{d}=0$

$\Rightarrow \vec{b}\cdot\vec{d}+\lambda(\vec{c}\cdot\vec{d})=0$

$\Rightarrow (6+2+6)+\lambda(3-2+6)=0$

$\Rightarrow 14+7\lambda=0$

$\Rightarrow \lambda=-2$

$\vec{a}=\vec{b}-2\vec{c}=3\hat{j}-\hat{k}$

Now $\left[\vec{a}\ \vec{b}\ \vec{c}\right]+\left[\vec{a}\vec{b}\vec{d}\right]+\left[\vec{a}\ \vec{c}\ \vec{d}\right]$

$= 0-(\vec{a}\times\vec{d})\cdot\vec{b}-(\vec{a}\times\vec{d})\cdot\vec{c}$

$= -(\vec{a}\times\vec{d})\cdot(\vec{b}+\vec{c})$

$= -(\vec{a}\times\vec{d})\cdot(3\hat{i}+2\hat{k})$

$= -\begin{vmatrix}0 & 3 & -1\\ 3 & 2 & 6\\ 3 & 0 & 2\end{vmatrix}$

$= -42$

18. Let [x] denote the greatest integer less than or equal to x. Then, the value of $x \in R$ satisfying the equation $[e^x]^2 + [e^x + 1] - 3 = 0$ lie in the interval :

माना [x] महत्तम पूर्णांक $\leq x$ है। तो समीकरण $[e^x]^2 + [e^x + 1] - 3 = 0$ को संतुष्ट करने वाली सभी वास्तविक संख्याएं x, निम्न में से किस अन्तराल में है?

(1) [0,1/e] (2) [1,e) (3) $[\log_e 2, \log_e 3)$ (4) $[0,\log_e 2)$

Ans. **(4)**

Sol. $[e^x]^2 + [e^x+1] - 3=0$

$[e^x]^2 + [e^x] - 2=0$

Let $[e^x] = t$

$t^2 + t - 2 = 0$

$(t+2)(t-1) = 0$

$t= 1, -2$

$[e^x]$: 1, –2 (–2 is not possible)

$[e^x] = 1$

$x\in[0, \ell n2)$

19. Let y = y(x) be the solution of the differential equation $\text{cosec}^2x\, dy + 2dx = (1 + y\cos 2x)\,\text{cosec}^2x\, dx$, with $y\left(\frac{\pi}{4}\right) = 0$, then the value of $(y(0) + 1)^2$ is equal to :

माना अवकल समीकरण $\text{cosec}^2x\, dy + 2dx = (1 + y\cos 2x)\,\text{cosec}^2x\, dx$, जबकि $y\left(\frac{\pi}{4}\right) = 0$ है, का हल y = y(x) है।

तो $(y(0) + 1)^2$ का मान बराबर है:

(1) $e^{1/2}$ (2) e^{-1} (3) e (4) $e^{-1/2}$

Ans. **(2)**

Sol. $\operatorname{cosec}^2 x dy + 2dx = (1+y\cos 2x)\operatorname{cosec}^2 x\, dx$

$$\frac{dy}{dx} + 2\sin^2 x = (1 + y\cos 2x)$$

$$\frac{dy}{dx} - y\cos 2x = 1 - 2\sin^2 x$$

$$\frac{dy}{dx} = \cos 2x(1+y)$$

$$\int \frac{dy}{(1+y)} = \int \cos 2x dx$$

$$\log(1+y) = \frac{\sin 2x}{2} + c$$

Given $y\left(\frac{\pi}{4}\right) = 0$

$$\log\left(1 + y\left(\frac{\pi}{4}\right)\right) = \frac{\sin\frac{\pi}{2}}{2} + c$$

$$c = \frac{-1}{2}$$

Now $\log(1 + y(0)) = \frac{\sin 0}{2} - \frac{1}{2}$

$$(1+y(0)) = e^{\frac{-1}{2}}$$

$$(1+y(0))^2 = e^{-1}$$

20. If the domain of the function $f(x) = \dfrac{\cos^{-1}\sqrt{x^2 - x + 1}}{\sqrt{\sin^{-1}\left(\frac{2x-1}{2}\right)}}$ is the interval $(\alpha, \beta]$, then $\alpha + \beta$ is equal to :

यदि फलन, $f(x) = \dfrac{\cos^{-1}\sqrt{x^2 - x + 1}}{\sqrt{\sin^{-1}\left(\frac{2x-1}{2}\right)}}$ का प्रान्त, अन्तराल $(\alpha, \beta]$, है तो $\alpha + \beta$ बराबर है:

(1) 2 (2) $\frac{3}{2}$ (3) $\frac{1}{2}$ (4) 1

Ans. **(2)**

Sol. $0 \le x^2 - x + 1 \le 1$ and $0 < \frac{2x-1}{2} \le 1$

$\Rightarrow \quad x(x-1) \le 0 \quad \& \quad 1 < 2x \le 3$

$\Rightarrow \quad x \in [0,1] \cap x \in \left(\frac{1}{2}, \frac{3}{2}\right]$

$\Rightarrow \quad x \in \left(\frac{1}{2}, 1\right]$

Hence $\alpha + \beta = \frac{1}{2} + 1 = \frac{3}{2}$

INTEGER QUESTIONS

1. Let A = {0,1,2,3,4,5,6,7} Then the number of bijective functions $f : A \to A$ such that f(1) + f(2) = 3 – f(3) is equal to -

माना : A = {0,1,2,3,4,5,6,7} एक समुच्चय है। तो फलनों $f : A \to A$ जो आच्छादक तथा एकैकी दोनों है तथा f(1) + f(2) = 3 – f(3) को संतुष्ट करते है, की संख्या बराबर है –

Ans. **(720)**

Sol. f(1) + f(2) = 3 – f(3)

$\Rightarrow$ f(1) + f(2) + f(3) = 3

$\Rightarrow$ {(f(1),f(2),f(3)} = {(0,1,2) (0,2,1) (1,0,2)(1,2,0)(2,1,0)(2,0,1)} = 3! = 6

and {f(0), f(4), f(5), f(6), f(7)} = 5!

Total such function = 5! × 3! = 720

2. Consider the following frequency distribution :

Class :	0-6	6-12	12-18	18-24	24-30
Frequency :	a	b	12	9	5

If mean = $\frac{309}{22}$ and median = 14, then the value $(a - b)^2$ is equal to.

निम्न बारंबारता बंटन पर विचार कीजिए:

वर्ग :	0-6	6-12	12-18	18-24	24-30
बारंबारता :	a	b	12	9	5

यदि इसका माध्य $\frac{309}{22}$ तथा माध्यिका 14 है, तो $(a - b)^2$ बराबर है ______ ।

Ans. **(4)**

Sol.

Midpoint	frequency	cumulativefreq.
3	a	a
9	b	a + b
15	12	a + b + 12
21	9	a + b + 21
27	5	a + b + 26
		n = a + b + 26

$$\text{mean} = \frac{3a + 9b + 180 + 189 + 135}{a + b + 26} = \frac{309}{22}$$

$\Rightarrow$ 81a + 37b = 1018 ....(1)

$$\text{median} = L + \frac{\frac{n}{2} - cf}{f} \times h$$

$$14 = 12 + \frac{\frac{a+b}{2} + 13 - (a+b)}{12} \times 6$$

$\Rightarrow$ a + b = 18 .....(2)

Solving (1) & (2), we get a = 8, b = 10

$\Rightarrow$ $(a - b)^2 = 4$

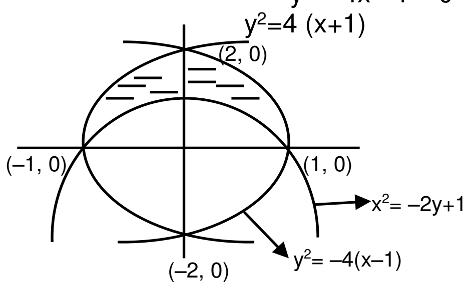
3. The area (in sq. units) of the region bounded by the curves $x^2 + 2y - 1 = 0$, $y^2 + 4x - 4 = 0$ and $y^2 - 4x - 4 = 0$, in the upper half plane is :

ऊपरी आधे निर्देशांक तल में वक्रों $x^2 + 2y - 1 = 0$, $y^2 + 4x - 4 = 0$ तथा $y^2 - 4x - 4 = 0$ द्वारा परिबद्ध क्षेत्र का क्षेत्रफल (वर्ग इकाइयों में) है__________।

Ans. **(2)**

Sol. $x^2 + 2y - 1 = 0$ $\qquad$ $y^2 + 4x - 4 = 0$

$y^2 - 4x - 4 = 0$



Area of common region is

$$A = 2\int_0^1 \left(\sqrt{4-4x} - \frac{1-x^2}{2}\right)dx \qquad \text{let } 4 - 4x = t^2$$

$$-4dx = 2tdt$$

$$A = \left[2\int_2^0 t\left(\frac{-t}{2}\right)dt - \left[\frac{x}{2} - \frac{x^3}{6}\right]_0^1\right]$$

$$A = 2\int_0^2 \frac{t^2}{2}dt - 2\left(\frac{1}{2} - \frac{1}{6}\right)$$

$$= 2\frac{2^3}{6} - \frac{4}{6} = \frac{12}{6} = 2$$

4. Let $y = y(x)$ be the solution of the differential equation $\left((x+2)e^{\left(\frac{y+1}{x+2}\right)} + (y+1)\right)dx = (x + 2)\, dy$, $y(1) = 1$.

If the domain of $y = y(x)$ is an open interval (α, β), then $|\alpha + \beta|$ is equal to -

माना अवकल समीकरण $\left((x+2)e^{\left(\frac{y+1}{x+2}\right)} + (y+1)\right)dx = (x + 2)\, dy$, $y(1) = 1$ का हल, $y = y(x)$ है। यदि $y = y(x)$ का प्रान्त विवृत अन्तराल (α, β) है, तो $|\alpha + \beta|$ बराबर है –

Ans. **(4)**

Sol. $\left((x+2)e^{\frac{y+1}{x+2}} + (y+1)\right)dx = (x + 2)dy$

$x + 2 = X \quad \Rightarrow dx = dX$

$y + 1 = Y \quad \Rightarrow dy = dY$

$$\left(Xe^{\frac{Y}{X}} + Y\right)dX = XdY$$

$\frac{dY}{dX} = e^{\frac{Y}{X}} + \frac{Y}{X}$

Put Y = tX $\Rightarrow \frac{dY}{dX} = t + X\frac{dt}{dX}$

$t + X\frac{dt}{dX} = e^t + t$

$\Rightarrow e^{-t}\,dt = \frac{dX}{X}$

$\Rightarrow -e^{-t} = \ln|X| + \ln|c|$

$\Rightarrow \ln|cX| = -e^{-t}$

$\Rightarrow \ln(-\ln|cX|) = -t$

$y + 1 = -(x + 2)\ln(-\ln|c(x + 2)|)$

$\ln|c(x + 2)| < 0$

$|c(x + 2)| < 1 = -1 < c(x + 2) < 1$

Case -1 $c > 0$

$\frac{-1}{c} < x + 2 < \frac{1}{c}$

$\frac{-1}{c} - 2 < x < \frac{1}{c} - 2$

Domain : $\left(\frac{-1}{c} - 2, \frac{1}{c} - 2\right) = (\alpha + \beta) \Rightarrow |\alpha + \beta| = 4$

Case -2 $c < 0$

$\frac{1}{c} - 2 < x < \frac{-1}{c} - 2$

Domain : $\left(\frac{1}{c} - 2, \frac{-1}{c} - 2\right) = (\alpha + \beta) \Rightarrow |\alpha + \beta| = 4$

Hence $|\alpha + \beta| = 4$

5. If the constant term, in binomial expansion of $\left(2x^r + \frac{1}{x^2}\right)^{10}$ is 180, then r is equal to -

यदि $\left(2x^r + \frac{1}{x^2}\right)^{10}$ के द्विपद प्रसार में अचर पद 180 है, तो r बराबर है –

Ans. **(8)**

Sol. $T_{k+1} = {}^{10}C_k (2x^r)^{10-k} (x)^{-2k} \Rightarrow {}^{10}C_k\, 2^{(10-k)}.\, x^{10r-rk-2k}$

Now, $10r - rk - 2k = 0 \Rightarrow r = \frac{2k}{10-k}$

And ${}^{10}C_k (2)^{10-k} = 180 \Rightarrow k = 8$

$r = \frac{2 \times 8}{10 - 8} = 8$

6. Let $f : R \to R$ be a function defined as $f(x) = \begin{cases} 3\left(1-\frac{|x|}{2}\right) & \text{if } |x| \le 2 \\ 0 & \text{if } |x| > 2 \end{cases}$

Let $g : R \to R$ be given by $g(x) = f(x + 2) - f(x - 2)$. If n and m denote the number of points in R where g is not continuous and not differentiable, respectively, then n + m is equal to :

माना फलन $f : R \to R$ $f(x) = \begin{cases} 3\left(1-\frac{|x|}{2}\right) & \text{यदि } |x| \le 2 \\ 0 & \text{यदि } |x| > 2 \end{cases}$

द्वारा परिभाषित है। माना $g : R \to R$, $g(x) = f(x + 2) - f(x - 2)$ द्वारा परिभाषित है। यदि R के उन बिन्दुओं की संख्या जहाँ g संतत नहीं है और जहाँ g अवकलनीय नहीं है, क्रमशः n और m है, तो n + m बराबर है –

Ans. (4)

Sol. $f(x) = \begin{cases} 3\left(1-\frac{|x|}{2}\right) & \text{if } |x| \le 2 \\ 0 & \text{if } |x| > 2 \end{cases}$

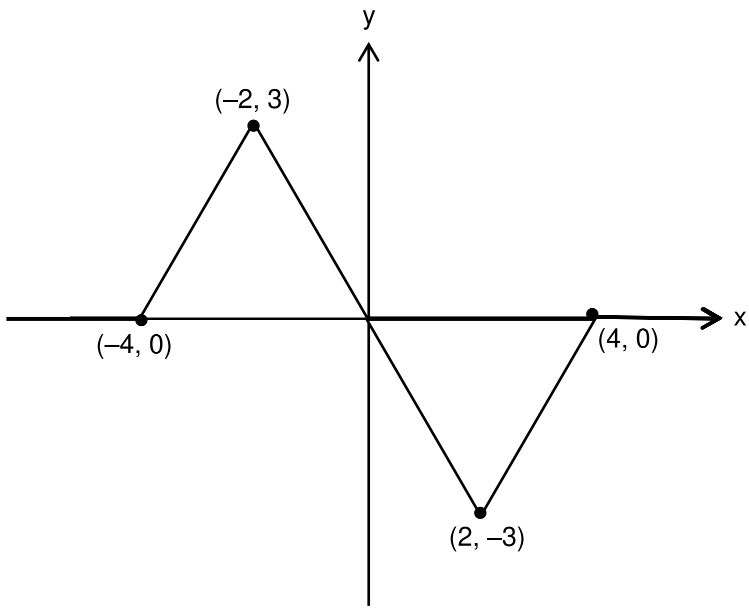
So, $f(x+2) = \begin{cases} 3\left(1-\frac{|x+2|}{2}\right) & \text{if } |x+2| \le 2 \\ 0 & \text{if } |x+2| > 2 \end{cases}$

$f(x+2) = \begin{cases} 3\left(1-\frac{|x+2|}{2}\right) & \text{if } x \in [-4, 0] \\ 0 & \text{if } x \in (-\infty, -4) \cup (0, \infty) \end{cases}$

Similarly $f(x-2) = \begin{cases} 3\left(1-\frac{|x-2|}{2}\right) & \text{if } x \in [0, 4] \\ 0 & \text{if } x \in (-\infty, 0) \cup (4, \infty) \end{cases}$

$g(x) = f(x + 2) - f(x - 2)$

So, $g(x) = \begin{cases} 3\left(1-\frac{|x+2|}{2}\right) & ; \quad x \in [-4, 0] \\ -3\left(1-\frac{|x-2|}{2}\right) & ; \quad x \in [0, 4] \\ 0 & ; \quad x \in (-\infty, 4) \cup (4, \infty) \end{cases}$



number of discontinuous points (n) = 0

Number of non-differentiable point (m) = 4

n + m = 4

7. The number of elements in the set $\{n \in \{1,2,3,........,100\} | (11)^n > (10)^n + (9)^n\}$ is :

समुच्च $\{n \in (1,2,3,........,100\} | (11)^n > (10)^n + (9)^n\}$ में अवयवों की संख्या है –

Ans. **(96)**

Sol. Let $11^n > 10^n + 9^n$ $\quad n \in \{1,2,3,\_\_\_\_\ 100\}$

$\Rightarrow 11^n - 9^n > 10^n$

$\Rightarrow (10+1)^n - (10-1)^n > 10^n$

$\Rightarrow 2\ [{}^nC_1 10^{n-1} + {}^nC_3 10^{n-3} + {}^nC_5 10^{n-5} + ...] > 10^n$

$$\Rightarrow \frac{1}{5}[{}^nC_1 10^n + {}^nC_3 10^{n-2} + {}^nC_5 10^{n-4} + ...] > 10^n$$

$$\Rightarrow \frac{1}{5}[{}^nC_1 + {}^nC_3 10^{-2} + {}^nC_5 10^{-4} + ...] > 1$$

Clearly the above inequality is true for $n \geq 5$

For n = 4 we have $\frac{1}{5}\left[4 + \frac{4}{10^2}\right] = \frac{4}{5}\left(\frac{101}{100}\right) < 1,$ Rejected

Hence, number of such $n \in \{1,2,3,\_\_\_\_\ 100\}$ is equal to 96

8. The sum of all the elements in the set $\{n \in \{1,2,......... 100\} \mid$ H.C.F. of n and 2040 is 1} is equal to :

समुच्च $\{n \in [1,2,......... 100\}|$ n तथा 2040 का महत्तम समापवर्तक 1 है} के सभी अवयवों का योग बराबर है ______I

Ans. **(1251)**

Sol. $2040 = 2^3.3^1.\ 5^1.\ 17$

Hence n cannot be multiple of 2,3,5 and 17

Then sum is n(1) – (n(2) + n(3) + n(5) + n(17) – n(6) – n(10) – n(34) – n(15) – n(51) – n(85)) + n(30))

Where n(a) means the sum of all numbers belonging to the set {1,2,......100} which are divisible by a

$$= \frac{100\times101}{2} - \left(\frac{2\times50\times51}{2} + \frac{3\times33\times34}{2} + \frac{5\times20\times21}{2} + \frac{17\times5\times6}{2} - \frac{6\times16\times17}{2} - \frac{10\times10\times11}{2} - \frac{34\times2\times3}{2} - \frac{15\times6\times7}{2} - 51 - 85 + 180\right)$$

= 5050 – 2550 – 1683 – 1050 – 255 + 816 + 550 + 102 + 315 + 51 + 85 – 180

= 1251

9. If the digits are not allowed to repeat in any number formed by using the digits 0,2,4,6,8,then the number of all numbers greater than 10,000 is equal to :

यदि अंको 0,2,4,6, तथा 8 द्वारा संख्याएँ बनाई गई है और उनमें अंकों को दोहराने की अनुमति नहीं है तो इस प्रकार बनाई गई उन संख्याओं, जो 10,000 से बड़ी हों की संख्या है ________I

Ans. **(96)**

Sol. Total number = $4 \times 4 \times 3 \times 2 \times 1 = 96$

10. Let $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Then the number of 3×3 matrices B with entries from the set {1,2,3,4,5} and satisfying AB = BA is -

माना $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ एक 3×3 आव्यूह है। तो आव्यूहों B जिनकी प्रविष्टियाँ, समुच्चय, {1,2,3,4,5} से हैं तथा जो AB = BA को संतुष्ट करते है, की संख्या है –

Ans. **(3125)**

Sol. $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Let $B = \begin{bmatrix} a & b & c \\ p & q & r \\ x & y & z \end{bmatrix}$

Now AB = BA

$$\Rightarrow \begin{bmatrix} p & q & r \\ a & b & c \\ x & y & z \end{bmatrix} = \begin{bmatrix} b & a & c \\ q & p & r \\ y & x & z \end{bmatrix}$$

$\Rightarrow p = b,\ a = q,\ r = c,\ x = y,\ \&\ z = z$

Hence number of such matrices are $5^5 = 3125$