PHYSICS

SECTION-A

1. Which of the following statements are correct?

(A) Electric monopoles do not exist whereas magnetic monopoles exist.

(B) Magnetic field lines due to a solenoid at its ends and outside cannot be completely straight and confined.

(C) Magnetic field lines are completely confined within a toroid.

(D) Magnetic field lines inside a bar magnet are not parallel.

(E) $\chi = -1$ is the condition for a perfect diamagnetic material, where χ is its magnetic susceptibility.

Choose the correct answer from the options given below :

(1) (C) and (E) only

(2) (B) and (D) only

(3) (A) and (B) only

(4) (B) and (C) only

Official Ans. by NTA (1)

Sol. Statement (C) is correct because, the magnetic field outside the toroid is zero and they form closed loops inside the toroid itself.

Statement (E) is correct because we know that super conductors are materials inside which the net magnetic field is always zero and they are perfect diamagnetic.

$\mu_r = 1 + \chi$

$\chi = -1$

$\mu_r = 0$

For superconductors.

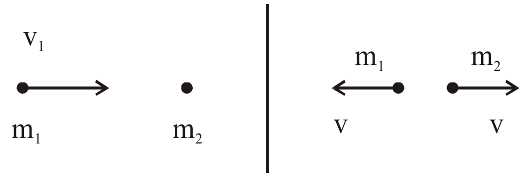
2. An object of mass m_1 collides with another object of mass m_2, which is at rest. After the collision the objects move with equal speeds in opposite direction. The ratio of the masses $m_2 : m_1$ is :

(1) 3 : 1 (2) 2 : 1

(3) 1 : 2 (4) 1 : 1

Official Ans. by NTA (1)

Sol.



$$m_1v_1 = -m_1v + m_2v$$

$$v_1 = -v + \frac{m_2}{m_1}v$$

$$\frac{(v_1 + v)}{v} = \frac{m_2}{m_1}$$

$$e = \frac{2v}{v_1} = 1$$

$$v = \frac{v_1}{2}$$

$$\frac{v_1 + v_1/2}{v_1/2} = \frac{m_2}{m_1}$$

$$3 = \frac{m_2}{m_1}$$

3. For an adiabatic expansion of an ideal gas, the fractional change in its pressure is equal to (where γ is the ratio of specific heats):

(1) $-\gamma\frac{dV}{V}$ (2) $-\gamma\frac{V}{dV}$

(3) $-\frac{1}{\gamma}\frac{dV}{V}$ (4) $\frac{dV}{V}$

Official Ans. by NTA (1)

Sol. PV^{γ} = constant

Differentiating

$$\frac{dP}{dV} = -\frac{\gamma P}{V}$$

$$\frac{dP}{P} = -\frac{\gamma dV}{V}$$

4. A proton and an α-particle, having kinetic energies K_p and K_α, respectively, enter into a magnetic field at right angles.
The ratio of the radii of trajectory of proton to that of α-particle is 2 : 1. The ratio of $K_p : K_\alpha$ is :

(1) 1 : 8 (2) 8 : 1
(3) 1 : 4 (4) 4 : 1

Official Ans. by NTA (4)

Sol. $r = \frac{mv}{qB} = \frac{p}{qB}$ $\qquad \frac{m_\alpha}{m_p} = 4$

$$\frac{r_p}{r_\alpha} = \frac{p_p}{q_p}\frac{q_\alpha}{p_\alpha} = \frac{2}{1}$$

$$\frac{p_p}{p_\alpha} = \frac{2q_p}{q_\alpha} = 2\left(\frac{1}{2}\right)$$

$$\frac{p_p}{p_\alpha} = 1$$

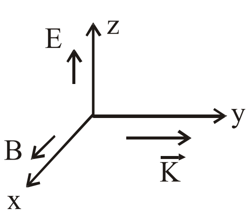
$$\frac{K_p}{K_\alpha} = \frac{p_p^2}{p_\alpha^p}\frac{m_\alpha}{m_p} = (1)\ (4)$$

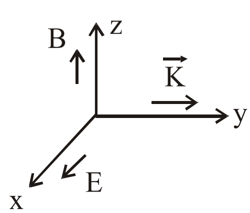
5. A plane electromagnetic wave propagating along y-direction can have the following pair of electric field $(\vec{E})$ and magnetic field $(\vec{B})$ components.

(1) E_y, B_y or E_z, B_z
(2) E_y, B_x or E_x, B_y
(3) E_x, B_z or E_z, B_x
(4) E_x, B_y or E_y, B_x

Official Ans. by NTA (3)

Sol.





6. Consider a uniform wire of mass M and length L. It is bent into a semicircle. Its moment of inertia about a line perpendicular to the plane of the wire passing through the centre is :

(1) $\frac{1}{4}\frac{ML^2}{\pi^2}$ (2) $\frac{2}{5}\frac{ML^2}{\pi^2}$

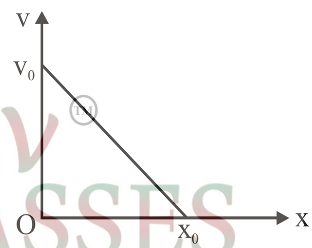
(3) $\frac{ML^2}{\pi^2}$ (4) $\frac{1}{2}\frac{ML^2}{\pi^2}$

Official Ans. by NTA (3)

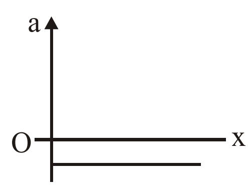
Sol. $\pi r = L \Rightarrow r = \frac{L}{\pi}$

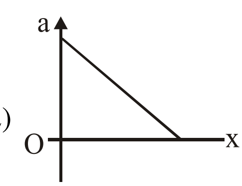
$$I = Mr^2 = \frac{ML^2}{\pi^2}$$

7. The velocity-displacement graph of a particle is shown in the figure.



The acceleration-displacement graph of the same particle is represented by :

(1)


(2)


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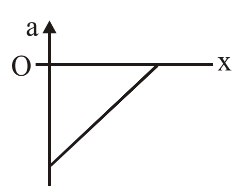

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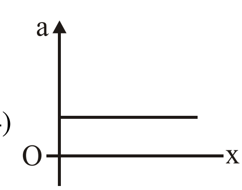


Official Ans. by NTA (3)

Sol. $v = -\left(\frac{v_0}{x_0}\right)x + v_0$

$$a = \frac{vdv}{dx}$$

$$a = \left[-\left(\frac{v_0}{x_0}\right)x + v_0\right]\left[-\frac{v_0}{x_0}\right]$$

$$a = \left(\frac{v_0}{x_0}\right)^2 x - \frac{v_0^2}{x_0}$$

8. The correct relation between α (ratio of collector current to emitter current) and β (ratio of collector current to base current) of a transistor is :

(1) $\beta = \frac{\alpha}{1+\alpha}$

(2) $\alpha = \frac{\beta}{1-\alpha}$

(3) $\beta = \frac{1}{1-\alpha}$

(4) $\alpha = \frac{\beta}{1+\beta}$

Official Ans. by NTA (4)

Sol. $\alpha = \frac{I_C}{I_E}, \beta = \frac{I_C}{I_B}$

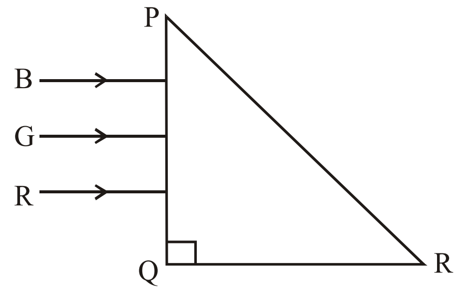
$I_E = I_B + I_C$

$$\alpha = \frac{I_C}{I_B + I_C} = \frac{1}{\frac{I_B}{I_C} + 1}$$

$$\alpha = \frac{1}{\frac{1}{\beta} + 1}$$

$$\alpha = \frac{\beta}{1+\beta}$$

9. Three rays of light, namely red (R), green (G) and blue (B) are incident on the face PQ of a right angled prism PQR as shown in figure.



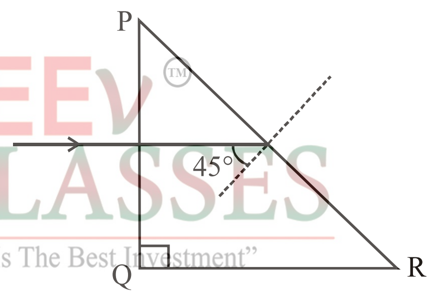
The refractive indices of the material of the prism for red, green and blue wavelength are 1.27, 1.42 and 1.49 respectively. The colour of the ray(s) emerging out of the face PR is :

(1) green (2) red

(3) blue and green (4) blue

Official Ans. by NTA (2)

Sol.



Assuming that the right angled prism is an isoceles prism, so the other angles will be 45° each.

⇒ Each incident ray will make an angle of 45° with the normal at face PR.

⇒ The wavelength corresponding to which the incidence angle is less than the critical angle, will pass through PR.

⇒ θ_C = critical angle

$$\Rightarrow \theta_C = \sin^{-1}\left(\frac{1}{\mu}\right)$$

⇒ If $\theta_C \geq 45°$

the light ray will pass

$$\Rightarrow (\theta_C)_{Red} = \sin^{-1}\left(\frac{1}{1.27}\right) = 51.94°$$

Red will pass.

$$\Rightarrow (\theta_C)_{Green} = \sin^{-1}\left(\frac{1}{1.42}\right) = 44.76°$$

Green will not pass

$$\Rightarrow (\theta_C)_{Blue} = \sin^{-1}\left(\frac{1}{1.49}\right) = 42.15°$$

Blue will not pass

⇒ So only red will pass through PR.

10. If the angular velocity of earth's spin is increased such that the bodies at the equator start floating, the duration of the day would be approximately :

(Take : $g = 10\ ms^{-2}$, the radius of earth, $R = 6400 \times 10^3$ m, Take $\pi = 3.14$)

(1) 60 minutes

(2) does not change

(3) 1200 minutes

(4) 84 minutes

Official Ans. by NTA (4)

Sol. For objects to float

$mg = m\omega^2R$

ω = angular velocity of earth.

R = Radius of earth

$$\omega = \sqrt{\frac{g}{R}} \qquad \ldots (1)$$

Duration of day = T

$$T = \frac{2\pi}{\omega} \qquad \ldots (2)$$

$$\Rightarrow T = 2\pi\sqrt{\frac{R}{g}}$$

$$= 2\pi\sqrt{\frac{6400 \times 10^3}{10}}$$

$$\Rightarrow \frac{T}{60} = 83.775 \text{ minutes}$$

$\simeq$ 84 minutes

11. The decay of a proton to neutron is :

(1) not possible as proton mass is less than the neutron mass

(2) possible only inside the nucleus

(3) not possible but neutron to proton conversion is possible

(4) always possible as it is associated only with β^+ decay

Official Ans. by NTA (2)

Sol. It is possible only inside the nucleus and not otherwise.

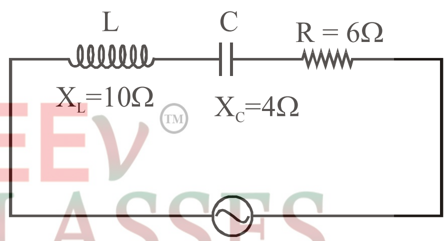
12. In a series LCR circuit, the inductive reactance (X_L) is 10 Ω and the capacitive reactance (X_C) is 4 Ω. The resistance (R) in the circuit is 6 Ω. The power factor of the circuit is :

(1) $\frac{1}{2}$ (2) $\frac{1}{2\sqrt{2}}$

(3) $\frac{1}{\sqrt{2}}$ (4) $\frac{\sqrt{3}}{2}$

Official Ans. by NTA (3)

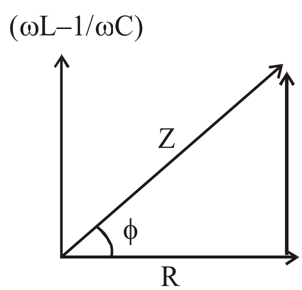
Sol.



We know that power factor is cosϕ,

$$\cos\phi = \frac{R}{Z} \qquad \ldots (1)$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \qquad \ldots (2)$$



$$\Rightarrow Z = \sqrt{6^2 + (10-4)^2}$$

$$\Rightarrow Z = 6\sqrt{2} \ \mid\ \cos\phi = \frac{6}{6\sqrt{2}}$$

$$\cos\phi = \frac{1}{\sqrt{2}}$$

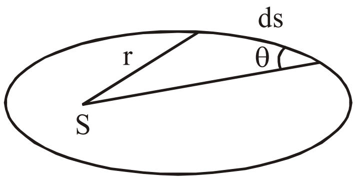
13. The angular momentum of a planet of mass M moving around the sun in an elliptical orbit is $\vec{L}$. The magnitude of the areal velocity of the planet is :

(1) $\frac{4L}{M}$ (2) $\frac{L}{M}$

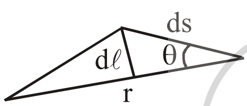
(3) $\frac{2L}{M}$ (4) $\frac{L}{2M}$

Official Ans. by NTA (4)

Sol.



For small displacement ds of the planet its area can be written as



$$dA = \frac{1}{2} r d\ell$$

$$= \frac{1}{2} r ds \sin\theta$$

$$\text{A.vel} = \frac{dA}{dt} = \frac{1}{2} r \sin\theta \frac{ds}{dt} = \frac{Vr\sin\theta}{2}$$

$$\frac{dA}{dt} = \frac{1}{2}\frac{mVr\sin\theta}{m} = \frac{L}{2m}$$

14. The function of time representing a simple harmonic motion with a period of $\frac{\pi}{\omega}$ is :

(1) $\sin(\omega t) + \cos(\omega t)$

(2) $\cos(\omega t) + \cos(2\omega t) + \cos(3\omega t)$

(3) $\sin^2(\omega t)$

(4) $3\cos\left(\frac{\pi}{4} - 2\omega t\right)$

Official Ans. by NTA (4)

Sol. Time period $T = \frac{2\pi}{\omega'}$

$$\frac{\pi}{\omega} = \frac{2\pi}{\omega'}$$

$\omega' = 2\omega \rightarrow$ Angular frequency of SHM

Option (3)

$$\sin^2\omega t = \frac{1}{2}\left(2\sin^2\omega t\right) = \frac{1}{2}\left(1 - \cos 2\omega t\right)$$

Angular frequency of $\left(\frac{1}{2} - \frac{1}{2}\cos 2\omega t\right)$ is 2ω

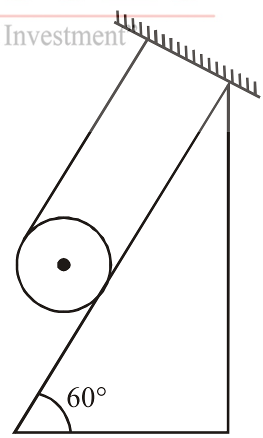
Option (4)

Angular frequency of SHM

$3\cos\left(\frac{\pi}{4} - 2\omega t\right)$ is 2ω.

So option (3) & (4) both have angular frequency 2ω but option (4) is direct answer.

15. A solid cylinder of mass m is wrapped with an inextensible light string and, is placed on a rough inclined plane as shown in the figure. The frictional force acting between the cylinder and the inclined plane is :



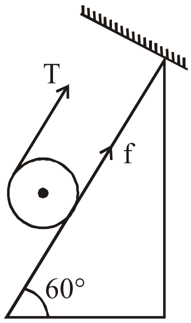
[The coefficient of static friction, μ_s, is 0.4]

(1) $\frac{7}{2}mg$ (2) 5 mg

(3) $\frac{mg}{5}$ (4) 0

Official Ans. by NTA (3)

Sol.



Let's take solid cylinder is in equilibrium

T + f = mg sin60 ....(i)

TR – fR = 0 ....(ii)

Solving we get

$$T = f_{req} = \frac{mg\sin\theta}{2}$$

But limiting friction < required friction

$$\mu mg\cos 60° < \frac{mg\sin 60°}{2}$$

∴ Hence cylinder will not remain in equilibrium

Hence f = kinetic

$= \mu_k N$

$= \mu_k mg\cos 60°$

$$= \frac{mg}{5}$$

16. The time taken for the magnetic energy to reach 25% of its maximum value, when a solenoid of resistance R, inductance L is connected to a battery, is :

(1) $\frac{L}{R}\ell n5$ (2) infinite

(3) $\frac{L}{R}\ell n2$ (4) $\frac{L}{R}\ell n10$

Official Ans. by NTA (3)

Sol. Magnetic energy = $\frac{1}{2}Li^2 = 25\%$

$$ME \Rightarrow 25\% \Rightarrow i = \frac{i_0}{2}$$

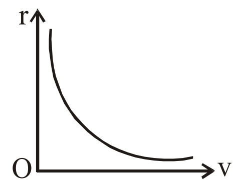
$i = i_0(1 - R^{-Rt/L})$ for charging

$$t = \frac{L}{R}\ell n2$$

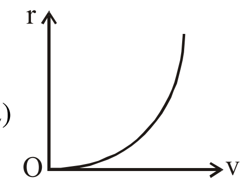
17. A particle of mass m moves in a circular orbit under the central potential field, $U(r) = \frac{-C}{r}$, where C is a positive constant.

The correct radius – velocity graph of the particle's motion is :

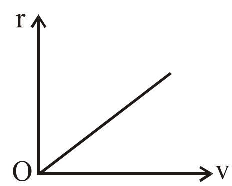
(1)



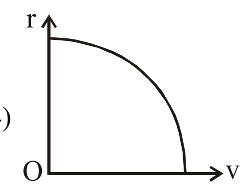
(2)



(3)



(4)



Official Ans. by NTA (1)

Sol. $U = -\frac{C}{r}$

$$F = -\frac{dU}{dr} = -\frac{C}{r^2}$$

$$|F| = \frac{mv^2}{r}$$

$$\frac{C}{r^2} = \frac{mv^2}{r}$$

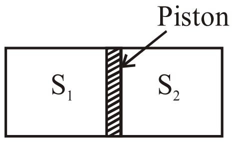
$$v^2 \propto \frac{1}{r}$$

18. An ideal gas in a cylinder is separated by a piston in such a way that the entropy of one part is S_1 and that of the other part is S_2. Given that $S_1 > S_2$. If the piston is removed then the total entropy of the system will be :

(1) $S_1 \times S_2$ (2) $S_1 - S_2$

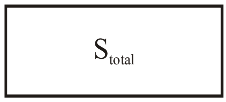
(3) $\frac{S_1}{S_2}$ (4) $S_1 + S_2$

Official Ans. by NTA (4)

Sol. 

; $S_1 > S_2$

After piston is removed



; $S_{total} = S_1 + S_2$

19. Consider a sample of oxygen behaving like an ideal gas. At 300 K, the ratio of root mean square (rms) velocity to the average velocity of gas molecule would be :
(Molecular weight of oxygen is 32 g/mol; $R = 8.3\ J\ K^{-1}\ mol^{-1}$)

(1) $\sqrt{\frac{3}{3}}$ (2) $\sqrt{\frac{8}{3}}$

(3) $\sqrt{\frac{3\pi}{8}}$ (4) $\sqrt{\frac{8\pi}{3}}$

Official Ans. by NTA (3)

Sol. $v_{rms} = \sqrt{\frac{3RT}{M}}$

$v_{avg} = \sqrt{\frac{8}{\pi}\frac{RT}{M}}$

$\frac{v_{rms}}{v_{avg}} = \sqrt{\frac{3\pi}{8}}$

20. The speed of electrons in a scanning electron microscope is $1 \times 10^7\ ms^{-1}$. If the protons having the same speed are used instead of electrons, then the resolving power of scanning proton microscope will be changed by a factor of:

(1) 1837 (2) $\frac{1}{1837}$

(3) $\sqrt{1837}$ (4) $\frac{1}{\sqrt{1837}}$

Official Ans. by NTA (1)

Sol. Resolving power (RP) $\propto \frac{1}{\lambda}$

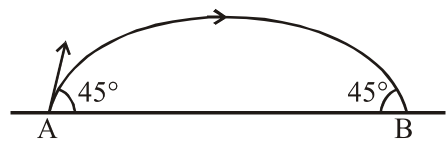
$\lambda = \frac{h}{P} = \frac{h}{mv}$

So (RP) $\propto \frac{mv}{h}$

RP $\propto$ P
RP $\propto$ mv
RP $\propto$ m

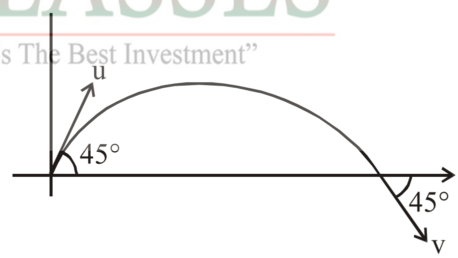
SECTION-B

1. The projectile motion of a particle of mass 5 g is shown in the figure.



The initial velocity of the particle is $5\sqrt{2}\ ms^{-1}$ and the air resistance is assumed to be negligible. The magnitude of the change in momentum between the points A and B is $x \times 10^{-2}\ kgms^{-1}$. The value of x, to the nearest integer, is _______.

Official Ans. by NTA (5)

Sol. 

$|\vec{u}| = |\vec{v}|$ ... (1)

$\vec{u} = u\cos 45\hat{i} + u\sin 45\hat{j}$ ... (2)

$\vec{v} = v\cos 45\hat{i} - v\sin 45\hat{j}$ ... (3)

$|\overline{\Delta P}| = |m(\vec{v} - \vec{u})|$ ... (4)

$\Delta P = 2mu \sin 45°$

$= 2 \times 5 \times 10^{-3} \times 5\sqrt{2} \times \frac{1}{\sqrt{2}}$

$= 50 \times 10^{-3}$
$= 5 \times 10^{-2}$

2. A ball of mass 4 kg, moving with a velocity of 10 ms^{-1}, collides with a spring of length 8 m and force constant 100 Nm^{-1}. The length of the compressed spring is x m. The value of x, to the nearest integer, is_____.

Official Ans. by NTA (6)

Sol. Let's say the compression in the spring by : y. So, by work energy theorem we have

$$\Rightarrow \frac{1}{2}mv^2 = \frac{1}{2}ky^2$$

$$\Rightarrow y = \sqrt{\frac{m}{k}} \cdot v$$

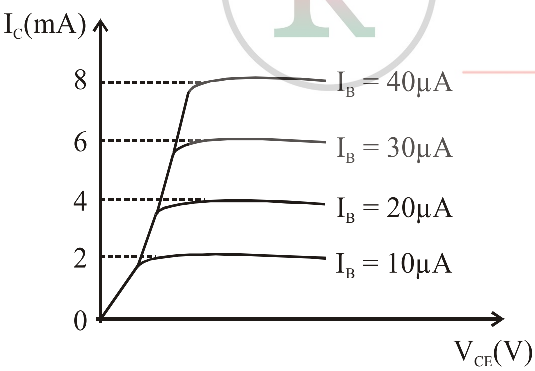
$$\Rightarrow y = \sqrt{\frac{4}{100}} \times 10$$

$\Rightarrow$ y = 2m

$\Rightarrow$ final length of spring

= 8 – 2 = 6m

3. The typical output characteristics curve for a transistor working in the common-emitter configuration is shown in the figure.



The estimated current gain from the figure is

Official Ans. by NTA (200)

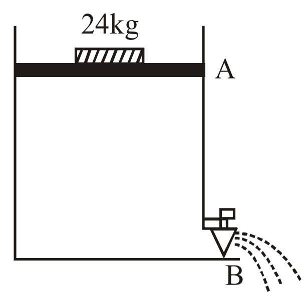
Sol. $\beta = \frac{\Delta I_C}{\Delta I_B} = \frac{2\times 10^{-3}}{10\times 10^{-6}}$

$$\beta = \frac{1}{5}\times 10^3$$

$\beta = 2 \times 10^2$

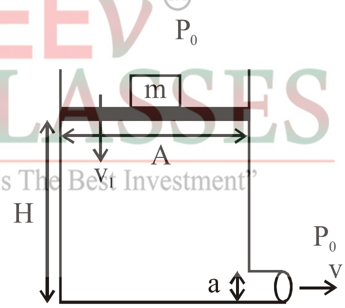
$\beta = 200$

4. Consider a water tank as shown in the figure. It's cross-sectional area is 0.4 m^2. The tank has an opening B near the bottom whose cross-section area is 1 cm^2. A load of 24 kg is applied on the water at the top when the height of the water level is 40 cm above the bottom, the velocity of water coming out the opening B is v ms^{-1}. The value of v, to the nearest integer, is___. [Take value of g to be 10 ms^{-2}]



Official Ans. by NTA (3)

Sol.



m = 24 kg

A = 0.4 m^2

a = 1 cm^2

H = 40cm

Using Bernoulli's equation

$$\Rightarrow \left(P_0 + \frac{mg}{A}\right) + \rho gH + \frac{1}{2}\rho v_1^2 \;(\to 0)$$

$$= P_0 + 0 + \frac{1}{2}\rho v^2 \quad \text{... (1)}$$

$\Rightarrow$ Neglecting v_1

$$\Rightarrow \quad v=\sqrt{2gH+\frac{2mg}{A\rho}}$$

$$\Rightarrow \quad v=\sqrt{8+1.2}$$

$\Rightarrow$ v = 3.033 m/s

$$\Rightarrow \quad v \simeq 3\,m/s$$

5. A TV transmission tower antenna is at a height of 20 m. Suppose that the receiving antenna is at.

(i) ground level

(ii) a height of 5 m.

The increase in antenna range in case (ii) relative to case (i) is n%.

The value of n, to the nearest integer, is .

Official Ans. by NTA (50)

Sol. Range = $\sqrt{2Rh}$

Range (i) = $\sqrt{2Rh}$

Range (ii) = $\sqrt{2Rh}+\sqrt{2Rh'}$

where h = 20 m & h' = 5m

$$\text{Ans} = \frac{\sqrt{2Rh'}}{\sqrt{2Rh}}\times 100\% = \frac{\sqrt{5}}{\sqrt{20}}\times 100\% = 50\%$$

6. The radius of a sphere is measured to be (7.50 ± 0.85) cm. Suppose the percentage error in its volume is x. The value of x, to the nearest x, is_______.

Official Ans. by NTA (34)

Sol. $\because\ v = \frac{4}{3}\pi r^3$

taking log & then differentiate

$$\frac{dV}{V}=3\frac{dr}{r}$$

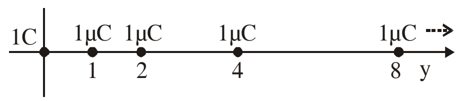
$$= \frac{3\times 0.85}{7.5}\times 100\% = 34\ \%$$

7. An infinite number of point charges, each carrying 1 μC charge, are placed along the y-axis at y = 1 m, 2 m, 4 m, 8 m................. The total force on a 1 C point charge, placed at the origin, is x × 10^3 N. The value of x, to the nearest integer, is________.

[Take $\frac{1}{4\pi \in_0}$ = 9 × 10^9 Nm^2/C^2]

Official Ans. by NTA (12)

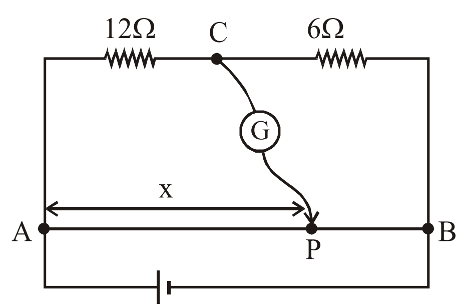
Sol.



$$F = k(1C)(1\mu C)\left[1+\frac{1}{2^2}+\frac{1}{4^2}+\frac{1}{8^2}+...\right]$$

$$= 9\times 10^3\left[\frac{1}{1-\frac{1}{4}}\right] = 12\times 10^3 N$$

8. Consider a 72 cm long wire AB as shown in the figure. The galvanometer jockey is placed at P on AB at a distance x cm from A. The galvanometer shows zero deflection.



The value of x, to the nearest integer, is

Official Ans. by NTA (48)

Sol. In Balanced conditions

$$\frac{12}{6}=\frac{x}{72-x}$$

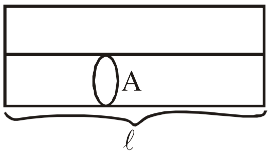
x = 48 cm

9. Two wires of same length and thickness having specific resistances 6Ω cm and 3Ω cm respectively are connected in parallel. The effective resistivity is ρ Ω cm. The value of ρ, to the nearest integer, is_____.

Official Ans. by NTA (4)

Sol. ∵ in parallel

$$R_{net} = \frac{R_1R_2}{R_1+R_2}$$



$$\frac{\rho\ell}{2A} = \frac{\rho_1\frac{\ell}{A}\times\rho_2\frac{\ell}{A}}{\rho_1\frac{\ell}{A}+\rho_2\frac{\ell}{A}}$$

$$\frac{\rho}{2} = \frac{6\times3}{6+3} = 2$$

$\rho = 4$

10. A galaxy is moving away from the earth at a speed of 286 kms^{-1}. The shift in the wavelength of a red line at 630 nm is $x \times 10^{-10}$ m. The value of x, to the nearest integer, is_____.
[Take the value of speed of light c, as 3×10^8 ms^{-1}]

Official Ans. by NTA (6)

Sol. $\frac{\Delta\lambda}{\lambda}c = v$

$$\Delta\lambda = \frac{v}{c}\times\lambda = \frac{286}{3\times10^5}\times630\times10^{-9} = 6\times10^{-10}$$

CHEMISTRY

SECTION-A

1. The oxidation states of nitrogen in NO, NO_2, N_2O and NO_3^- are in the order of :

(1) $NO_3^- > NO_2 > NO > N_2O$

(2) $NO_2 > NO_3^- > NO > N_2O$

(3) $N_2O > NO_2 > NO > NO_3^-$

(4) $NO > NO_2 > N_2O > NO_3^-$

Official Ans. by NTA (1)

Sol. The oxidation states of Nitrogen in following molecules are as follows

$NO_3^- \rightarrow +5$

$NO_2 \rightarrow +4$

$NO \rightarrow +2$

$N_2O \rightarrow +1$

2. In basic medium, H_2O_2 exhibits which of the following reactions ?

(A) $Mn^{2+} \rightarrow Mn^{4+}$

(B) $I_2 \rightarrow I^-$

(C) $PbS \rightarrow PbSO_4$

Choose the most appropriate answer from the options given below :

(1) (A), (C) only

(2) (A) only

(3) (B) only

(4) (A), (B) only

Official Ans. by NTA (4)

Sol. In basic medium, oxidising action of H_2O_2.

$Mn^{2+} + H_2O_2 \rightarrow Mn^{+4} + 2OH^-$

In basic medium, reducing action of H_2O_2

$I_2 + H_2O_2 + 2OH^- \rightarrow 2I^- + 2H_2O + O_2$

In acidic medium, oxidising action of H_2O_2.

$PbS(s) + 4H_2O_2(aq) \rightarrow PbSO_4(s) + 4H_2O(\ell)$

Hence correct option (4)

3. In the reaction of hypobromite with amide, the carbonyl carbon is lost as :

(1) CO_3^{2-}

(2) HCO_3^-

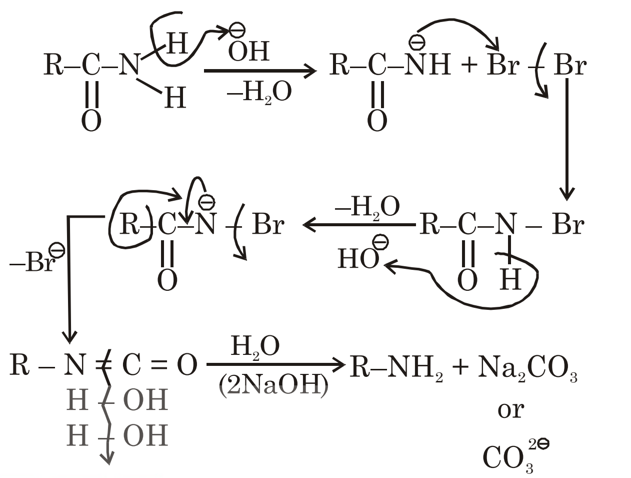
(3) CO_2

(4) CO

Official Ans. by NTA (1)

Sol. $R{-}\underset{\underset{O}{\|}}{C}{-}NH_2 + Br_2 + 4NaOH \rightarrow R{-}NH_2 + Na_2CO_3 + 2NaBr + 2H_2O$

Mechanism



4. The oxide that shows magnetic property is :

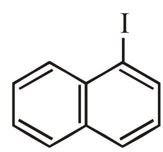
(1) SiO_2 (2) Mn_3O_4

(3) Na_2O (4) MgO

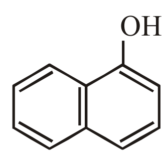
Official Ans. by NTA (2)

Sol. Mn_3O_4 shows magnetic properties.

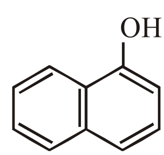
5. Main Products formed during a reaction of 1-methoxy naphthalene with hydroiodic acid are:

(1) 

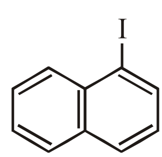
and CH_3OH

(2) 

and CH_3I

(3) 

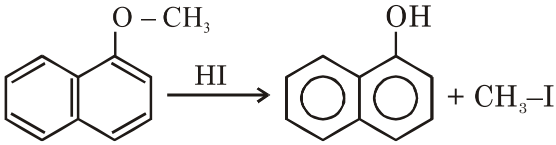
and CH_3OH

(4) 

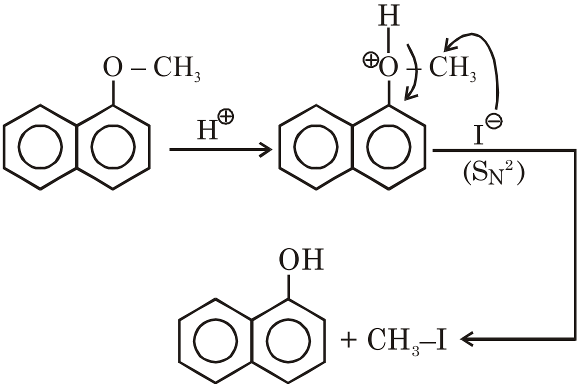
and CH_3I

Official Ans. by NTA (2)

Sol.



Mechanism



6. Deficiency of vitamin K causes :

(1) Increase in blood clotting time

(2) Increase in fragility of RBC's

(3) Cheilosis

(4) Decrease in blood clotting time

Official Ans. by NTA (1)

Sol. Due to deficiency of Vitmain K causes increases in blood clotting time.

Note : Vitamin K related to blood factor.

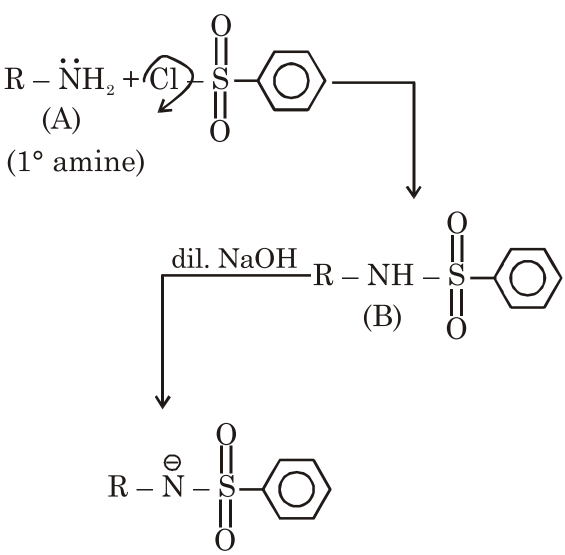
7. An organic compound "A" on treatment with benzene sulphonyl chloride gives compound B. B is soluble in dil. NaOH solution. Compound A is :

(1) $C_6H_5-N-(CH_3)_2$ (2) $C_6H_5-NHCH_2CH_3$

(3) $C_6H_5-CH_2\ NHCH_3$ (4) $C_6H_5-\underset{\substack{|\\CH_3}}{CH}-NH_2$

Official Ans. by NTA (4)

Sol. Hinsberg reagent (Benzene sulphonyl chloride) gives reaction product with 1° amine and it is soluble in dil. NaOH.



8. The first ionization energy of magnesium is smaller as compared to that of elements X and Y, but higher than that of Z. the elements X, Y and Z, respectively, are :

(1) chlorine, lithium and sodium

(2) argon, lithium and sodium

(3) argon, chlorine and sodium

(4) neon, sodium and chlorine

Official Ans. by NTA (3)

Sol. The 1st IE order of 3rd period is

Na < Al < Mg < Si < S < P < Cl < Ar

X & Y are Ar & Cl

Z is sodium (Na).

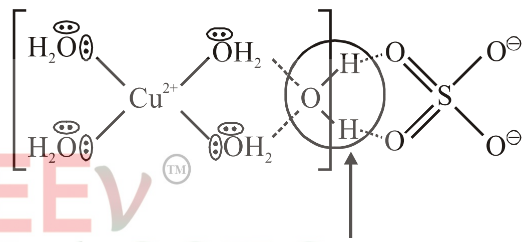
9. The secondary valency and the number of hydrogen bonded water molecule(s) in $CuSO_4{\cdot}5H_2O$, respectively, are :

(1) 6 and 4 (2) 4 and 1

(3) 6 and 5 (4) 5 and 1

Official Ans. by NTA (2)

Sol.



Hydrogen bonded water molecule = 1

Secondary valency = 4

10. Given below are two statements :

Statement I : Bohr's theory accounts for the stability and line spectrum of Li^+ ion.

Statement II : Bohr's theory was unable to explain the splitting of spectral lines in the presence of a magnetic field.

In the light of the above statements, choose the most appropriate answer from the options given below :

(1) Both statement I and statement II are true.

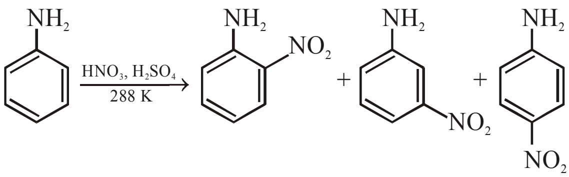
(2) Statement I is false but statement II is true.

(3) Both statement I and statement II are false.

(4) Statement I is true but statement II is false.

Official Ans. by NTA (2)

Sol. Statement-I is false since Bohr's theory accounts for the stability and spectrum of single electronic species (eg : He^+, Li^{2+} etc)
Statement II is true.

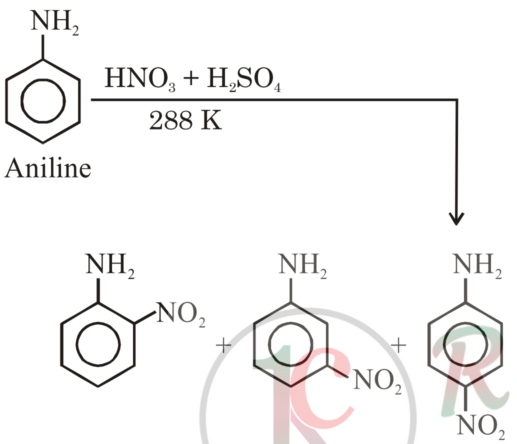
11. 

A B C

Consider the given reaction, percentage yield of :

(1) C > A > B (2) B > C > A
(3) A > C > B (4) C > B > A

Official Ans. by NTA (4)

Sol. 

(A) (2%) (B) (47%) (C) (51%)

% yield order ⇒ C > B > A

12. The charges on the colloidal CdS sol and TiO_2 sol are, respectively :

(1) positive and positive
(2) positive and negative
(3) negative and negative
(4) negative and positive

Official Ans. by NTA (4)

Sol. CdS sol → –ve sol
TiO_2 sol → +ve sol

13. Match List - I with List - II :

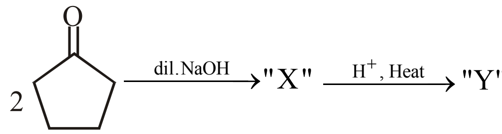
List - I	List - II
(Class of Chemicals)	(Example)
(a) Antifertility drug	(i) Meprobamate
(b) Antibiotic	(ii) Alitame
(c) Tranquilizer	(iii) Norethindrone
(d) Artificial Sweetener	(iv) Salvarsan

(1) (a)-(ii), (b)-(iii), (c)-(iv), (d)-(i)
(2) (a)-(iv), (b)-(iii), (c)-(ii), (d)-(i)
(3) (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii)
(4) (a)-(ii), (b)-(iv), (c)-(i), (d)-(iii)

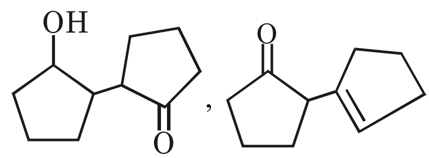
Official Ans. by NTA (3)

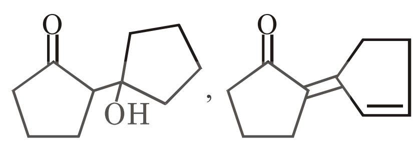
Sol. (A) Antifertility drug → (iii) Nor ethindrone
(B) Antibiotic → (iv) Salvarsan
(C) Tranquilizer → (i) Meprobamate
(D) Artificial sweetener → (ii) Alitame

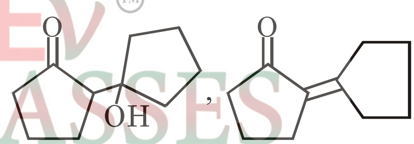
Ans. A–iii, B-iv, C-i, D-ii

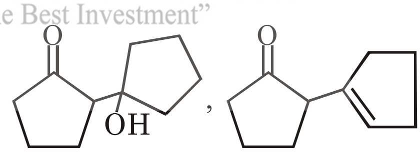
14. 

Consider the above reaction, the product 'X' and 'Y' respectively are :

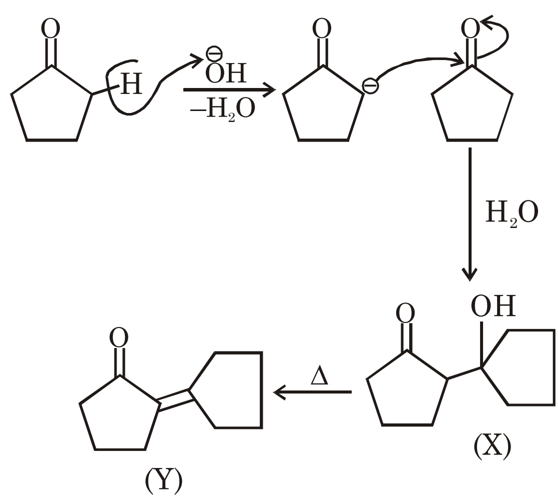
(1) 

(2) 

(3) 

(4) 

Official Ans. by NTA (3)

Sol. 

15. Match list-I with list-II :

List-I	**List-II**
(a) Be	(i) Treatment of cancer
(b) Mg	(ii) Extraction of metals
(c) Ca	(iii) Incendiary bombs and signals
(d) Ra	(iv) Windows of X-ray tubes
	(v) Bearings for motor engines.

Choose the most appropriate answer the option given below :

(1) a-iv, b-iii, c-i, d-ii
(2) a-iv, b-iii, c-ii, d-i
(3) a-iii, b-iv, c-v, d-ii
(4) a-iii, b-iv, c-ii, d-v

Official Ans. by NTA (2)

Sol. (a) Be → it is used in the Windows of X-ray tubes

(b) Mg → it is used in the Incendiary bombs and signals

(c) Ca → it is used in the Extraction of metals

(d) Ra → it is used in the Treatment of cancer

16. Given below are two statements :

Statement I : C_2H_5OH and AgCN both can generate nucleophile.

Statement II : KCN and AgCN both will generate nitrile nucleophile with all reaction conditions.

Choose the most appropriate option :

(1) Statement I is true but statement II is false
(2) Both statement I and statement II are true
(3) Statement I is false but statement II is true
(4) Both statement I and statement II are false

Official Ans. by NTA (1)

17. Given below are two statements :

Statement I : Non-biodegradable wastes are generated by the thermal power plants.

Statement II : Bio-degradable detergents leads to eutrophication.

In the light of the above statements, choose the most appropriate answer from the option given below :

(1) Both statement I and statement II are false
(2) Statement I is true but statement II is false
(3) Statement I is false but statement II is true
(4) Both statement I and statement II are true.

Official Ans. by NTA (4)

Sol. Non-biodegradable wastes are generated by the thermal power plants which produces fly ash. Detergents which are biodegradable causes problem called eutrophication which kills animal life by deprieving it of oxygen.

18. Match list-I with list-II :

List-I	**List-II**
(a) Mercury	(i) Vapour phase refining
(b) Copper	(ii) Distillation refining
(c) Silicon	(iii) Electrolytic refining
(d) Nickel	(iv) Zone refining

Choose the most appropriate answer from the option given below :

(1) a-i, b-iv, c-ii, d-iii
(2) a-ii, b-iii, c-i, d-iv
(3) a-ii, b-iii, c-iv, d-i
(4) a-ii, b-iv, c-iii, d-i

Official Ans. by NTA (3)

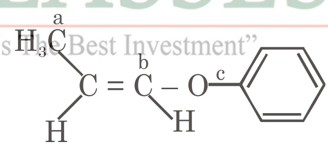
Sol. (a) Mercury → Distillation refining

(b) Copper → Electrolytic refining

(c) Silicon → Zone refining

(d) Nickel → Vapour phase refining

19. In the following molecules,

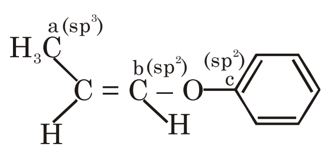


Hybridisation of carbon a, b and c respectively are :

(1) sp^3, sp, sp
(2) sp^3, sp^2, sp
(3) sp^3, sp^2, sp^2
(4) sp^3, sp, sp^2

Official Ans. by NTA (3)

Sol.



20. A hard substance melts at high temperature and is an insulator in both solid and in molten state. This solid is most likely to be a / an :

(1) Ionic solid
(2) Molecular solid
(3) Metallic solid
(4) Covalent solid

Official Ans. by NTA (4)

Sol. Covalent or network solid have very high melting point and they are insulators in their solid and molten form.

SECTION-B

1. A reaction has a half life of 1 min. The time required for 99.9% completion of the reaction is ______ min. (Round off to the Nearest integer)

[Use : ln 2= 0.69, ln 10 = 2.3]

Official Ans. by NTA (10)

Sol. $$\frac{t_{99.9\%}}{t_{50\%}} = \frac{\frac{1}{K}\ln\frac{100}{0.1}}{\frac{1}{K}\ln 2}$$

$$= \frac{\ln 1000}{\ln 2} \times t_{50\%}$$

$$= \frac{3\ln 10}{\ln 2} \times 1$$

$$= \frac{3 \times 2.3}{0.69} = 10$$

2. The molar conductivities at infinite dilution of barium chloride, sulphuric arid and hydrochloric acid are 280, 860 and 426 Scm^2 mol^{-1} respectively. The molar conductivity at infinite dilution of barium sulphate is ________S cm^2 mol^{-1}(Round off to the Nearest Integer).

Official Ans. by NTA (288)

Sol. From Kohlrausch's law

$$\Lambda_m^\infty(BaSO_4) = \lambda_m^\infty(Ba^{2+}) + \lambda_m^\infty(SO_4^{2-})$$

$$\Lambda_m^\infty(BaSO_4) = \Lambda_m^\infty(BaCl_2) + \Lambda_m^\infty(H_2SO_4) - 2\,\Lambda_m^\infty(HCl)$$

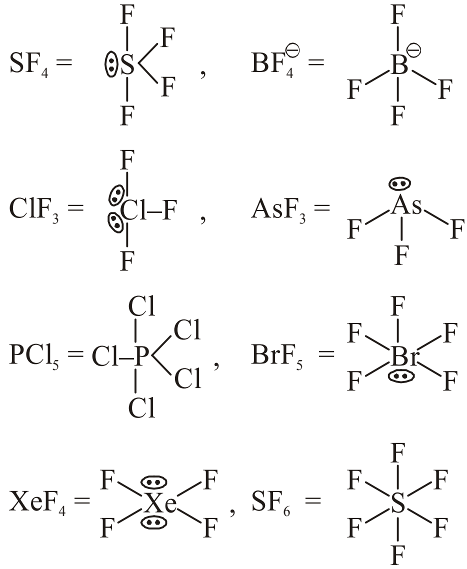
$= 280 + 860 - 2\ (426)$
$= 288\ Scm^2mol^{-1}$

3. The number of species below that have two lone pairs of electrons in their central atom is ____(Round off to the Nearest integer)

SF_4, BF_4^-, ClF_3, AsF_3, PCl_5, BrF_5, XeF_4, SF_6

Official Ans. by NTA (2)

Sol.

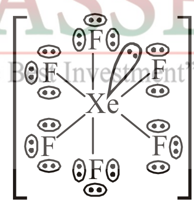


Two l.p. on central atom is = ClF_3, XeF_4

4. A xenon compound 'A' upon partial hydrolysis gives XeO_2F_2. The number of lone pair of electrons present in compound A is _______(Round off to the Nearest integer)

Official Ans. by NTA (19)

Sol. $XeF_6 + 2H_2O \longrightarrow XeO_2F_2 + 4HF$
(A) (Limited water)
Structure of 'A'



Total l.p. on (A) = 19

5. The gas phase reaction

$$2A(g) \rightleftharpoons A_2(g)$$

at 400 K has ΔG^o = + 25.2 kJ mol^{-1}.
The equilibrium constant K_C for this reaction is ______× 10^{-2}. (Round off to the Nearest integer)
[Use : R = 8.3 J mol^{-1}K^{-1} , ln 10 = 2.3
$\log_{10} 2$ = 0.30, 1 atm = 1 bar]
[antilog (–0.3) = 0.501]

Official Ans. by NTA (166)

Official Ans. by ETOOS (2) Sol.

Using formula

$\Delta_r G^0 = -RT\ln K_p$

$25200 = -2.3 \times 8.3 \times 400 \log(K_p)$

$K_p = 10^{-3.3} = 10^{-3} \times 0.501$

$= 5.01 \times 10^{-4}\ Bar^{-1}$

$= 5.01 \times 10^{-9}\ Pa^{-1}$

$= \dfrac{K_C}{8.3 \times 400}$

$K_C = 1.66 \times 10^{-5}\ m^3/mole$

$= 1.66 \times 10^{-2}$ L/mol

Ans = 2

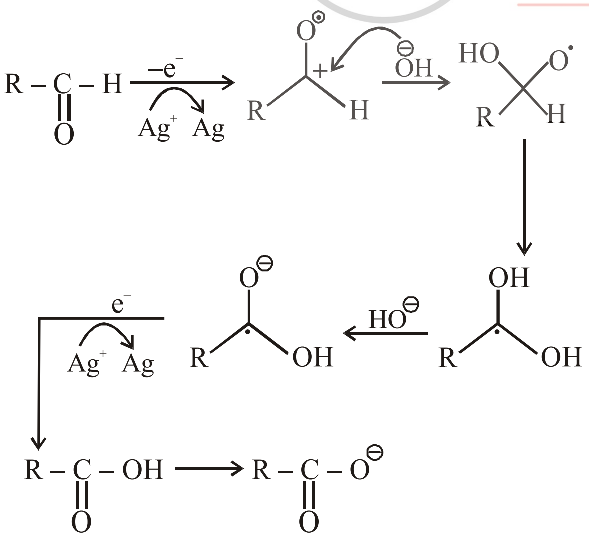
6. In Tollen's test for aldehyde, the overall number of electron(s) transferred to the Tollen's reagent formula $[Ag(NH_3)_2]^+$ per aldehyde group to form silver mirror is_______.(Round off to the Nearest integer)

Official Ans. by NTA (2)

Sol. $AgNO_3 + NaOH \rightarrow AgOH + NaNO_3$

$2AgOH \rightarrow Ag_2O + H_2O$

$Ag_2O + 4NH_3 + H_2O \rightarrow 2Ag(NH_3)_2^+ + 2\overset{\ominus}{O}H$



Total $2e^-$ transfer to Tollen's reagent

7. The solubility of $CdSO_4$ in water is 8.0×10^{-4} mol L^{-1}. Its solubility in 0.01 M H_2SO_4 solution is ______$\times 10^{-6}$ mol L^{-1}. (Round off to the Nearest integer) (Assume that solubility is much less than 0.01 M)

Official Ans. by NTA (64)

Sol. In pure water,

$K_{sp} = S^2 = (8 \times 10^{-4})^2$

$= 64 \times 10^{-8}$

In 0.01 M H_2SO_4

$H_2SO_{4(aq)} \rightarrow 2H^+_{(aq)} + SO_4^{2-}(aq.)$

0.02 0.01

$BaSO_{4(s)} \rightleftharpoons Ba^{2+}_{(aq.)} + SO_4^{2-}{}_{(aq)}$

x x (x + 0.01)

$K_{sp} = x\,(x + 0.01)$

$= 64 \times 10^{-8}$

$x + 0.01 \cong 0.01$ M

So, $x\,(0.01) = 64 \times 10^{-8}$

$x = 64 \times 10^{-6}$ M

8. A solute a dimerizes in water. The boiling point of a 2 molar solution of A is 100.52ºC. The percentage association of A is._____.

(Round off to the Nearest integer)

[Use : K_b for water = 0.52 K kg mol^{-1}

Boiling point of water = 100°C]

Official Ans. by NTA (50)

Official Ans. by ETOOS (100)

Sol. $\Delta T_b = T_b - T_b^0$

100.52 – 100

= 0.52ºC

$i = \left(1 - \dfrac{\alpha}{2}\right)$

$\because\ \Delta T_b = i\,K_b \times m$

$0.52 = \left(1 - \dfrac{\alpha}{2}\right) \times 0.52 \times 2$

$\alpha = 1$

So, percentage association = 100%.

9. 10.0 ml of Na_2CO_3 solution is titrated against 0.2 M HCl solution. The following titre values were obtained in 5 readings.

4.8 ml, 4.9 ml, 5.0 ml, 5.0 ml and 5.0 ml

Based on these readings, and convention of titrimetric estimation of concentration of Na_2CO_3 solution is ________mM.

(Round off to the Nearest integer)

Official Ans. by NTA (50)

Sol. Most precise volume of HCl = 5 ml at equivalence point

Meq. of Na_2CO_3 = meq. of HCl.

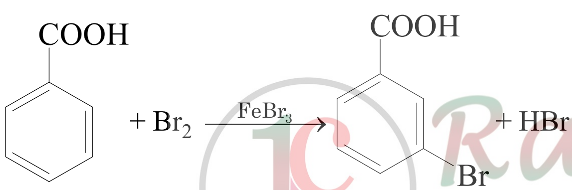
Let molarity of $Na2CO_3$ solution = M, then

$M \times 10 \times 2 = 0.2 \times 5 \times 1$

M = 0.05 mol / L

$= 0.05 \times 1000$

= 50 mM

10. 

Consider the above reaction where 6.1 g of benzoic acid is used to get 7.8 g of m-bromo benzoic acid. The percentage yield of the product is_____.

(Round off to the Nearest integer)

[Given : Atomic masses : C = 12.0u, H : 1.0u, O : 16.0u, Br = 80.0 u]

Official Ans. by NTA (78)

Sol. Moles of Benzoic acid $= \frac{6.1}{122}$

= moles of m-bromobenzoic acid

So, weight of m-bromobenzoic acid

$= \frac{6.1}{122} \times 201\text{gm}$

= 10.05 gm

$\%\ \text{yield} = \frac{\text{Actual weight}}{\text{Theoretical weight}} \times 100$

$= \frac{7.8}{10.05} \times 100$

= 77.61%

MATHEMATICS

SECTION-A

1. Let y = y(x) be the solution of the differential equation $\frac{dy}{dx} = (y+1)\left((y+1)e^{x^2/2} - x\right), 0 < x < 2.1,$ with y(2) = 0. Then the value of $\frac{dy}{dx}$ at x = 1 is equal to :

(1) $\frac{-e^{3/2}}{(e^2+1)^2}$ (2) $-\frac{2e^2}{(1+e^2)^2}$

(3) $\frac{e^{5/2}}{(1+e^2)^2}$ (4) $\frac{5e^{1/2}}{(e^2+1)^2}$

Official Ans. by NTA (1)

Sol. Let y + 1 = Y

$\therefore \frac{dY}{dx} = Y^2 e^{\frac{x^2}{2}} - xY$

Put $-\frac{1}{Y} = k$

$\Rightarrow \frac{dk}{dx} + k(-x) = e^{\frac{x^2}{2}}$

I.F. = $e^{-\frac{x^2}{2}}$

$\therefore k = (x+c)e^{x^2/2}$

Put $k = -\frac{1}{y+1}$

$\therefore y+1 = -\frac{1}{(x+c)e^{x^2/2}}$ ...(i)

when x = 2, y = 0, then c = $-2 - \frac{1}{e^2}$

diffentiate equation (i) & put x = 1

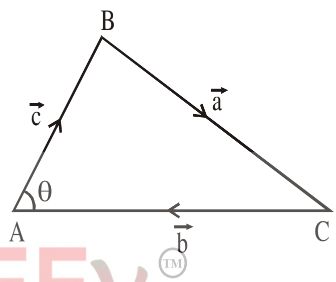
we get $\left(\frac{dy}{dx}\right)_{x=1} = -\frac{e^{3/2}}{(1+e^2)^2}$

2. In a triangle ABC, if $|\overrightarrow{BC}| = 8$, $|\overrightarrow{CA}| = 7$, $|\overrightarrow{AB}| = 10$, then the projection of the vector $\overrightarrow{AB}$ on $\overrightarrow{AC}$ is equal to :

(1) $\frac{25}{4}$ (2) $\frac{85}{14}$ (3) $\frac{127}{20}$ (4) $\frac{115}{16}$

Official Ans. by NTA (2)

Sol.



$|\vec{a}| = 8, |\vec{b}| = 7, |\vec{c}| = 10$

$\cos\theta = \frac{|\vec{b}|^2 + |\vec{c}|^2 - |\vec{a}|^2}{2|\vec{b}||\vec{c}|} = \frac{17}{28}$

Projection of $\vec{c}$ on $\vec{b}$

$= |\vec{c}|\cos\theta$

$= 10 \times \frac{17}{28}$

$= \frac{85}{14}$

3. Let the system of linear equations

4x + λy + 2z = 0

2x – y + z = 0

μx + 2y + 3z = 0, λ, μ ∈ R.

has a non-trivial solution. Then which of the following is true ?

(1) μ = 6, λ ∈ R (2) λ = 2, μ ∈ R

(3) λ = 3, μ ∈ R (4) μ = –6, λ ∈ R

Official Ans. by NTA (1)

Sol. For non-trivial solution

$$\begin{vmatrix} 4 & \lambda & 2 \\ 2 & -1 & 1 \\ \mu & 2 & 3 \end{vmatrix} = 0$$

$\Rightarrow 2\mu - 6\lambda + \lambda\mu = 12$

when $\mu = 6$, $12 - 6\lambda + 6\lambda = 12$

which is satisfied by all λ

4. Let $f : R - \{3\} \to R - \{1\}$ be defined by $f(x) = \frac{x-2}{x-3}$. Let $g : R \to R$ be given as $g(x) = 2x - 3$. Then, the sum of all the values of x for which $f^{-1}(x) + g^{-1}(x) = \frac{13}{2}$ is equal to

(1) 7 (2) 2 (3) 5 (4) 3

Official Ans. by NTA (3)

Sol. $f(x) = y = \frac{x-2}{x-3}$

$\therefore x = \frac{3y-2}{y-1}$

$\therefore f^{-1}(x) = \frac{3x-2}{x-1}$

& $g(x) = y = 2x - 3$

$\therefore x = \frac{y+3}{2}$

$\therefore g^{-1}(x) = \frac{x+3}{2}$

$\because f^{-1}(x) + g^{-1}(x) = \frac{13}{2}$

$\therefore x^2 - 5x + 6 = 0$ ⟨ x_1, x_2

$\therefore$ sum of roots

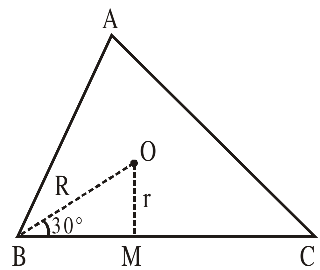
$x_1 + x_2 = 5$

5. Let the centroid of an equilateral triangle ABC be at the origin. Let one of the sides of the equilateral triangle be along the straight line $x + y = 3$. If R and r be the radius of circumcircle and incircle respectively of ΔABC, then $(R + r)$ is equal to :

(1) $\frac{9}{\sqrt{2}}$ (2) $7\sqrt{2}$ (3) $2\sqrt{2}$ (4) $3\sqrt{2}$

Official Ans. by NTA (1)

Sol.



$r = OM = \frac{3}{\sqrt{2}}$

& $\sin 30° = \frac{1}{2} = \frac{r}{R} \Rightarrow R = \frac{6}{\sqrt{2}}$

$\therefore r + R = \frac{9}{\sqrt{2}}$

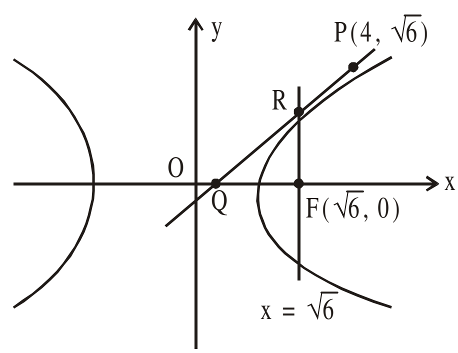
6. Consider a hyperbola $H : x^2 - 2y^2 = 4$. Let the tangent at a point $P(4, \sqrt{6})$ meet the x-axis at Q and latus rectum at $R(x_1, y_1)$, $x_1 > 0$. If F is a focus of H which is nearer to the point P, then the area of ΔQFR is equal to

(1) $4\sqrt{6}$ (2) $\sqrt{6} - 1$

(3) $\frac{7}{\sqrt{6}} - 2$ (4) $4\sqrt{6} - 1$

Official Ans. by NTA (3)

Sol.



$\frac{x^2}{4}-\frac{y^2}{2}=1$

$e=\sqrt{1+\frac{b^2}{a^2}}=\sqrt{\frac{3}{2}}$

$\therefore$ Focus F(ae, 0) $\Rightarrow$ $F\left(\sqrt{6},0\right)$

equation of tangent at P to the hyperbola is

$2x - y\sqrt{6} = 2$

tangent meet x-axis at Q(1, 0)

& latus rectum $x = \sqrt{6}$ at $R\left(\sqrt{6},\frac{2}{\sqrt{6}}\left(\sqrt{6}-1\right)\right)$

$\therefore$ Area of Δ_{QFR} = $\frac{1}{2}\left(\sqrt{6}-1\right).\frac{2}{\sqrt{6}}\left(\sqrt{6}-1\right)$

$=\frac{7}{\sqrt{6}}-2$

7. If P and Q are two statements, then which of the following compound statement is a tautology ?

(1) $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow Q$
(2) $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow \sim P$
(3) $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow P$
(4) $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow (P \wedge Q)$

Official Ans. by NTA (2)

Sol. LHS of all the options are some i.e.

$((P \rightarrow Q) \wedge \sim Q)$

$\equiv (\sim P \vee Q) \wedge \sim Q$

$\equiv (\sim P \wedge \sim Q) \vee (Q \wedge \sim Q)$

$\equiv \sim P \wedge \sim Q$

(A) $(\sim P \wedge \sim Q) \rightarrow Q$

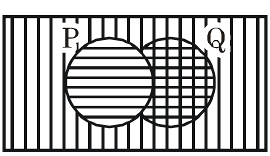
$\equiv \sim(\sim P \wedge \sim Q) \vee Q$

$\equiv (P \vee Q) \vee Q \neq$ tautology

(B) $(\sim P \wedge \sim Q) \rightarrow \sim P$

$\equiv \sim(\sim P \wedge \sim Q) \vee \sim P$

$\equiv (P \vee Q) \vee \sim P$



$\Rightarrow$ Tautology

(C) $(\sim P \wedge \sim Q) \rightarrow P$

$\equiv (P \vee Q) \vee P \neq$ Tautology

(D) $(\sim P \wedge \sim Q) \rightarrow (P \wedge Q)$

$\equiv (P \vee Q) \vee (P \wedge Q) \neq$ Tautology

Aliter :

P	Q	$P \vee Q$	$P \vee Q$	$\sim P$	$(P \vee Q) \vee \sim P$
T	T	T	T	F	T
T	F	T	F	F	T
F	T	T	F	T	T
F	F	F	F	T	T

8. Let $g(x)=\int_0^x f(t)\,dt$, where f is continuous function in [0, 3] such that $\frac{1}{3}\le f(t)\le 1$ for all $t \in [0, 1]$ and $0 \le f(t) \le \frac{1}{2}$ for all $t \in (1, 3]$. The largest possible interval in which g(3) lies is :

(1) $\left[-1,-\frac{1}{2}\right]$
(2) $\left[-\frac{3}{2},-1\right]$
(3) $\left[\frac{1}{3},2\right]$
(4) [1, 3]

Official Ans. by NTA (3)

Sol. $\frac{1}{3}\le f(t)\le 1\ \forall\ t\in[0,1]$

$0\le f(t)\le\frac{1}{2}\ \forall\ t\in(1,3]$

Now, $g(3)=\int_0^3 f(t)\,dt=\int_0^1 f(t)\,dt+\int_1^3 f(t)\,dt$

$\because \int_0^1 \frac{1}{3}\,dt \le \int_0^1 f(t)\,dt \le \int_0^1 1.\,dt$ ....(1)

and $\int_1^3 0\,dt \le \int_1^3 f(1)\,dt \le \int_1^3 \frac{1}{2}\,dt$ ....(2)

Adding, we get

$$\frac{1}{3}+0 \le g(3) \le 1+\frac{1}{2}(3-1)$$

$$\frac{1}{3} \le g(3) \le 2$$

9. Let S_1 be the sum of first 2n terms of an arithmetic progression. Let S_2 be the sum of first 4n terms of the same arithmetic progression. If $(S_2 - S_1)$ is 1000, then the sum of the first 6n terms of the arithmetic progression is equal to:

(1) 1000 (2) 7000 (3) 5000 (4) 3000

Official Ans. by NTA (4)

Sol. $S_{2n} = \frac{2n}{2}[2a + (2n - 1)d]$, $S_{4n} = \frac{4n}{2}[2a + (4n - 1)d]$

$\Rightarrow S_2 - S_1 = \frac{4n}{2}[2a + (4n - 1)d] - \frac{2n}{2}[2a + (2n - 1)d]$

$= 4an + (4n - 1)2nd - 2na - (2n - 1)dn$

$= 2na + nd[8n - 2 - 2n + 1]$

$\Rightarrow 2na + 2n[6n - 1] = 1000$

$2a + (6n - 1)d = \frac{1000}{n}$

Now, $S_{6n} = \frac{6n}{2}[2a+(6n-1)d]$

$= 3n.\frac{1000}{n} = 3000$

10. Let a complex number be $w = 1 - \sqrt{3}i$. Let another complex number z be such that $|zw| = 1$ and $\arg(z) - \arg(w) = \frac{\pi}{2}$. Then the area of the triangle with vertices origin, z and w is equal to :

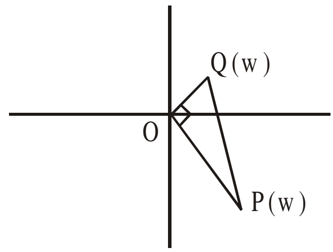
(1) 4 (2) $\frac{1}{2}$ (3) $\frac{1}{4}$ (4) 2

Official Ans. by NTA (2)

Sol. $w = 1 - \sqrt{3}.i \Rightarrow |w| = 2$

Now, $|z| = \frac{1}{|w|} \Rightarrow |z| = \frac{1}{2}$

and $\text{amp}(z) = \frac{\pi}{2} + \text{amp}(w)$



$\Rightarrow$ Area of triangle $= \frac{1}{2}.OP.OQ$

$= \frac{1}{2}.2.\frac{1}{2} = \frac{1}{2}$

11. Let in a series of 2n observations, half of them are equal to a and remaining half are equal to –a. Also by adding a constant b in each of these observations, the mean and standard deviation of new set become 5 and 20, respectively. Then the value of $a^2 + b^2$ is equal to :

(1) 425 (2) 650 (3) 250 (4) 925

Official Ans. by NTA (1)

Sol. Let observations are denoted by x_i for $1 \le i < 2n$

$$\bar{x} = \frac{\sum x_i}{2n} = \frac{(a+a+...+a)-(a+a+...+a)}{2n}$$

$\Rightarrow \bar{x} = 0$

and $\sigma_x^2 = \frac{\sum x_i^2}{2n} - (\bar{x})^2 = \frac{a^2+a^2+...+a^2}{2n} - 0 = a^2$

$\Rightarrow \sigma_x = a$

Now, adding a constant b then $\bar{y} = \bar{x} + b = 5$

$\Rightarrow b = 5$

and $\sigma_y = \sigma_x$ (No change in S.D.) $\Rightarrow a = 20$

$\Rightarrow a^2 + b^2 = 425$

12. Let $S_1 : x^2 + y^2 = 9$ and $S_2 : (x-2)^2 + y^2 = 1$. Then the locus of center of a variable circle S which touches S_1 internally and S_2 externally always passes through the points :

(1) $\left(0, \pm\sqrt{3}\right)$ (2) $\left(\frac{1}{2}, \pm\frac{\sqrt{5}}{2}\right)$

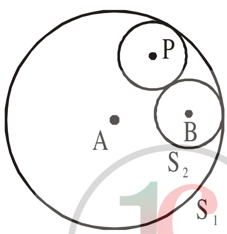
(3) $\left(2, \pm\frac{3}{2}\right)$ (4) $\left(1, \pm 2\right)$

Official Ans. by NTA (3)

Sol. $S_1 : x^2 + y^2 = 9 \begin{cases} r_1 = 3 \\ A(0, 0) \end{cases}$

$S_2 : (x-2)^2 + y^2 = 1 \begin{cases} r_2 = 1 \\ B(2, 0) \end{cases}$

$\because c_1c_2 = r_1 - r_2$



$\therefore$ given circle are touching internally

Let a veriable circle with centre P and radius r

$\Rightarrow PA = r_1 - r$ and $PB = r_2 + r$

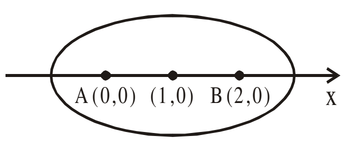
$\Rightarrow PA + PB = r_1 + r_2$

$\Rightarrow PA + PB = 4 \quad (> AB)$

$\Rightarrow$ Locus of P is an ellipse with foci at A(0, 0) and B(2, 0) and length of major axis is 2a = 4,

$e = \frac{1}{2}$

$\Rightarrow$ centre is at (1, 0) and $b^2 = a^2(1 - e^2) = 3$ if x-ellipse



$\Rightarrow E : \frac{(x-1)^2}{4} + \frac{y^2}{3} = 1$

which is satisfied by $\left(2, \pm\frac{3}{2}\right)$

13. Let $\vec{a}$ and $\vec{b}$ be two non-zero vectors perpendicular to each other and $|\vec{a}| = |\vec{b}|$. If $|\vec{a} \times \vec{b}| = |\vec{a}|$, then the angle between the vectors $\left(\vec{a} + \vec{b} + \left(\vec{a} \times \vec{b}\right)\right)$ and $\vec{a}$ is equal to :

(1) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (2) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

(3) $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$ (4) $\sin^{-1}\left(\frac{1}{\sqrt{6}}\right)$

Official Ans. by NTA (2)

Sol. $|\vec{a}| = |\vec{b}|$, $|\vec{a} \times \vec{b}| = |\vec{a}|$, $\vec{a} \perp \vec{b}$

$|\vec{a} \times \vec{b}| = |\vec{a}| \Rightarrow |\vec{a}||\vec{b}|\sin 90^\circ = |\vec{a}| \Rightarrow |\vec{b}| = 1 = |\vec{a}|$

$\vec{a}$ and $\vec{b}$ are mutually perpendicular unit vectors.

Let $\vec{a} = \hat{i}$, $\vec{b} = \hat{j} \Rightarrow \vec{a} \times \vec{b} = \hat{k}$

$\cos\theta = \frac{\left(\hat{i} + \hat{j} + \hat{k}\right).\hat{i}}{\sqrt{3}\sqrt{1}} = \frac{1}{\sqrt{3}} \Rightarrow \theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

14. Let in a Binomial distribution, consisting of 5 independent trials, probabilities of exactly 1 and 2 successes be 0.4096 and 0.2048 respectively. Then the probability of getting exactly 3 successes is equal to :

(1) $\frac{32}{625}$ (2) $\frac{80}{243}$ (3) $\frac{40}{243}$ (4) $\frac{128}{625}$

Official Ans. by NTA (1)

Sol. $P(X = 1) = {}^5C_1.p.q^4 = 0.4096$

$P(X = 2) = {}^5C_2.p^2.q^3 = 0.2048$

$\Rightarrow \frac{q}{2p} = 2$

$\Rightarrow q = 4p$ and $p + q = 1$

$\Rightarrow p = \frac{1}{5}$ and $q = \frac{4}{5}$

Now

$P(X = 3) = {}^5C_3.\left(\frac{1}{5}\right)^3.\left(\frac{4}{5}\right)^2 = \frac{10 \times 16}{125 \times 25} = \frac{32}{625}$

15. Let a tangent be drawn to the ellipse $\frac{x^2}{27}+y^2=1$ at $\left(3\sqrt{3}\cos\theta, \sin\theta\right)$ where $\theta \in \left(0, \frac{\pi}{2}\right)$. Then the value of θ such that the sum of intercepts on axes made by this tangent is minimum is equal to :

(1) $\frac{\pi}{8}$ (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{6}$ (4) $\frac{\pi}{3}$

Official Ans. by NTA (3)

Sol. Equation of tangent be

$$\frac{x\cos\theta}{3\sqrt{3}}+\frac{y.\sin\theta}{1}=1, \qquad \theta \in \left(0,\frac{\pi}{2}\right)$$

intercept on x-axis

OA = $3\sqrt{3}\sec\theta$

intercept on y-axis

OB = $\text{cosec}\theta$

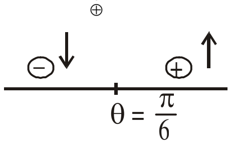
Now, sum of intercept

= $3\sqrt{3}\sec\theta + \text{cosec}\theta = f(\theta)$ let

$f'(\theta) = 3\sqrt{3}\sec\theta\tan\theta - \text{cosec}\theta\cot\theta$

$$=3\sqrt{3}\frac{\sin\theta}{\cos^2\theta}-\frac{\cos\theta}{\sin^2\theta}$$

$$=\underbrace{\frac{\cos\theta}{\sin^2\theta}.3\sqrt{3}}_{\oplus}\left[\tan^2\theta-\frac{1}{3\sqrt{3}}\right]=0 \Rightarrow \theta=\frac{\pi}{6}$$



$\Rightarrow$ at $\theta = \frac{\pi}{6}$, $f(\theta)$ is minimum

16. Define a relation R over a class of n × n real matrices A and B as "ARB iff there exists a non-singular matrix P such that $PAP^{-1} = B$". Then which of the following is true ?

(1) R is symmetric, transitive but not reflexive,
(2) R is reflexive, symmetric but not transitive
(3) R is an equivalence relation
(4) R is reflexive, transitive but not symmetric

Official Ans. by NTA (3)

Sol. A and B are matrices of n × n order & ARB iff there exists a non singular matrix $P(\det(P) \neq 0)$ such that $PAP^{-1} = B$

For reflexive

ARA $\Rightarrow PAP^{-1} = A$ ...(1) must be true

for P = I, Eq.(1) is true so 'R' is reflexive

For symmetric

ARB $\Leftrightarrow PAP^{-1} = B$ ...(1) is true

for BRA iff $PBP^{-1} = A$ ...(2) must be true

$\because PAP^{-1} = B$

$P^{-1}PAP^{-1} = P^{-1}B$

$IAP^{-1}P = P^{-1}BP$

$A = P^{-1}BP$ ...(3)

from (2) & (3) $PBP^{-1} = P^{-1}BP$

can be true some $P = P^{-1} \Rightarrow P^2 = I$ $(\det(P) \neq 0)$

So 'R' is symmetric

For trnasitive

ARB $\Leftrightarrow PAP^{-1} = B$... is true

BRC $\Leftrightarrow PBP^{-1} = C$... is true

now $PPAP^{-1}P^{-1} = C$

$P^2A(P^2)^{-1} = C \Rightarrow ARC$

So 'R' is transitive relation

$\Rightarrow$ Hence R is equivalence

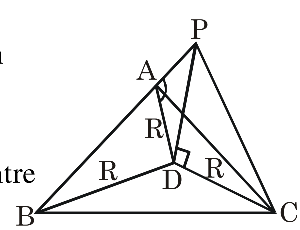
17. A pole stands vertically inside a triangular park ABC. Let the angle of elevation of the top of the pole from each corner of the park be $\frac{\pi}{3}$. If the radius of the circumcircle ot ΔABC is 2, then the height of the pole is equal to :

(1) $\frac{2\sqrt{3}}{3}$ (2) $2\sqrt{3}$ (3) $\sqrt{3}$ (4) $\frac{1}{\sqrt{3}}$

Official Ans. by NTA (2)

Sol. Let PD = h, R = 2

As angle of elevation of top of pole from A, B, C are equal So D must be circumcentre of ΔABC



$$\tan\left(\frac{\pi}{3}\right)=\frac{PD}{R}=\frac{h}{R}$$

$$h=R\tan\left(\frac{\pi}{3}\right)=2\sqrt{3}$$

18. If $15\sin^4\alpha + 10\cos^4\alpha = 6$, for some $\alpha \in R$, then the value of $27\sec^6\alpha + 8\text{cosec}^6\alpha$ is equal to :

(1) 350 (2) 500 (3) 400 (4) 250

Official Ans. by NTA (4)

Sol. $15\sin^4\alpha + 10\cos^4\alpha = 6$

$15\sin^4\alpha + 10\cos^4\alpha = 6(\sin^2\alpha + \cos^2\alpha)^2$

$(3\sin^2\alpha - 2\cos^2\alpha)^2 = 0$

$$\tan^2\alpha = \frac{2}{3},\ \cot^2\alpha = \frac{3}{2}$$

$\Rightarrow 27\sec^6\alpha + 8\text{cosec}^6\alpha$

$= 27(\sec^6\alpha)^3 + 8(\text{cosec}^6\alpha)^3$

$= 27(1 + \tan^2\alpha)3 + 8(1 + \cot^2\alpha)^3$

$= 250$

19. The area bounded by the curve $4y^2 = x^2(4 - x)(x - 2)$ is equal to :

(1) $\frac{\pi}{8}$ (2) $\frac{3\pi}{8}$ (3) $\frac{3\pi}{2}$ (4) $\frac{\pi}{16}$

Official Ans. by NTA (3)

Sol. $4y^2 = x^2(4 - x)(x - 2)$

$$|y| = \frac{|x|}{2}\sqrt{(4-x)(x-2)}$$

$$\Rightarrow y_1 = \frac{x}{2}\sqrt{(4-x)(x-2)}$$

$$\text{and } y_2 = \frac{-x}{2}\sqrt{(4-x)(x-2)}$$

$D : x \in [2, 4]$

Required Area

$$= \int_2^4 (y_1 - y_2)\,dx = \int_2^4 x\sqrt{(4-x)(x-2)}\,dx \quad ...(1)$$

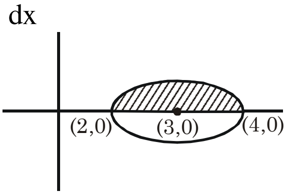
$$\text{Applying } \int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

$$\text{Area} = \int_2^4 (6-x)\sqrt{(4-x)(x-2)}\,dx \quad ...(2)$$

(1) + (2)

$$2A = 6\int_2^4 \sqrt{(4-x)(x-2)}\,dx$$

$$A = 3\int_2^4 \sqrt{1-(x-3)^2}\,dx$$



$$A = 3.\frac{\pi}{2}.1^2 = \frac{3\pi}{2}$$

20. Let $f : R \to R$ be a function defined as

$$f(x) = \begin{cases} \dfrac{\sin(a+1)x + \sin 2x}{2x} & , \text{if } x < 0 \\ b & , \text{if } x = 0 \\ \dfrac{\sqrt{x + bx^3} - \sqrt{x}}{bx^{5/2}} & , \text{if } x > 0 \end{cases}$$

If f is continuous at $x = 0$, then the value of $a + b$ is equal to :

(1) $-\frac{5}{2}$ (2) -2 (3) -3 (4) $-\frac{3}{2}$

Official Ans. by NTA (4)

Sol. $f(x)$ is continuous at $x = 0$

$$\lim_{x\to 0^+} f(x) = f(0) = \lim_{x\to 0^-} f(x) \quad ...(1)$$

$$f(0) = b \quad ...(2)$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+}\left(\frac{\sin(a+1)x}{2x} + \frac{\sin 2x}{2x}\right)$$

$$= \frac{a+1}{2} + 1 \quad ...(3)$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{\sqrt{x+bx^3} - \sqrt{x}}{bx^{5/2}}$$

$$= \lim_{x\to 0^+} \frac{(x + bx^3 - x)}{bx^{5/2}\left(\sqrt{x+bx^3} + \sqrt{x}\right)}$$

$$= \lim_{x\to 0^+} \frac{\sqrt{x}}{\sqrt{x}\left(\sqrt{1+bx^2} + 1\right)} = \frac{1}{2} \quad ...(4)$$

Use (2), (3) & (4) in (1)

$$\frac{1}{2} = b = \frac{a+1}{2} + 1$$

$$\Rightarrow b = \frac{1}{2},\ a = -2$$

$$a + b = \frac{-3}{2}$$

SECTION-B

1. If $f(x)$ and $g(x)$ are two polynomials such that the polynomial $P(x) = f(x^3) + xg(x^3)$ is divisible by $x^2 + x + 1$, then $P(1)$ is equal to______.

Official Ans. by NTA (0)

Sol. $P(x) = f(x^3) + xg(x^3)$

$P(1) = f(1) + g(1)$ ...(1)

Now $P(x)$ is divisible by $x^2 + x + 1$

$\Rightarrow P(x) = Q(x)(x^2 + x + 1)$

$P(w) = 0 = P(w^2)$ where w, w^2 are non-real cube roots of units

$P(x) = f(x^3) + xg(x^3)$

$P(w) = f(w^3) + wg(w^3) = 0$

$f(1) + wg(1) = 2$ ...(2)

$P(w^2) = f(w^6) + w^2g(w^6) = 0$

$f(1) + w^2g(1) = 0$ ...(3)

(2) + (3)

$\Rightarrow 2f(1) + (w + w^2)g(1) = 0$

$2f(1) = g(1)$ ...(4)

(2) – (3)

$\Rightarrow (w - w^2)g(1) = 0$

$g(1) = 0 = f(1)$ from (4)

from (1) $P(1) = f(1) + g(1) = 0$

2. Let I be an identity matrix of order 2 × 2 and $P = \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}$. Then the value of $n \in N$ for which $P^n = 5I - 8P$ is equal to _______.

Official Ans. by NTA (6)

Sol. $P = \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}$

$$5I - 8P = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 16 & -8 \\ 40 & -24 \end{bmatrix} = \begin{bmatrix} -11 & 8 \\ -40 & 29 \end{bmatrix}$$

$$P^2 = \begin{bmatrix} -1 & 1 \\ -5 & 4 \end{bmatrix}$$

$$P^3 = \begin{bmatrix} 3 & -2 \\ 10 & -7 \end{bmatrix} \Rightarrow P^6 = \begin{bmatrix} -11 & 8 \\ -40 & 29 \end{bmatrix} = P^n$$

$\Rightarrow n = 6$

3. If $\sum_{r=1}^{10} r!\left(r^3 + 6r^2 + 2r + 5\right) = \alpha(11!)$, then the value of α is equal to _______.

Official Ans. by NTA (160)

Sol. $$\sum_{r=1}^{10} r!\{(r+1)(r+2)(r+3) - 9(r+1) + 8\}$$

$$= \sum_{r=1}^{10} \left[\{(r+3)! - (r+1)!\} - 8\{(r+1)! - r!\}\right]$$

$= (13! + 12! - 2! - 3!) - 8(11! - 1)$

$= (12.13 + 12 - 8).11! - 8 + 8$

$= (160)(11)!$

Hence $\alpha = 160$

4. The term independent of x in the expansion of $\left[\frac{x+1}{x^{2/3} - x^{1/3} + 1} - \frac{x-1}{x - x^{1/2}}\right]^{10}$, $x \neq 1$, is equal to _______.

Official Ans. by NTA (210)

Sol. $$\left(\left(x^{1/3}+1\right) - \left(\frac{\sqrt{x}+1}{\sqrt{x}}\right)\right)^{10}$$

$(x^{1/3} - x^{-1/2})^{10}$

$T_{r+1} = {}^{10}C_r(x^{1/3})^{10-r}(-x^{-1/2})^r$

$\frac{10-r}{3} - \frac{r}{2} = 0 \Rightarrow 20 - 2r - 3r = 0$

$\Rightarrow r = 4$

$$T_5 = {}^{10}C_4 = \frac{10\times9\times8\times7}{4\times3\times2\times1} = 210$$

5. Let P(x) be a real polynomial of degree 3 which vanishes at $x = -3$. Let P(x) have local minima at $x = 1$, local maxima at $x = -1$ and $\int_{-1}^{1} P(x)\,dx = 18$, then the sum of all the coefficients of the polynomial P(x) is equal to _______.

Official Ans. by NTA (8)

Sol. Let $p'(x) = a(x - 1)(x + 1) = a(x^2 - 1)$

$p(x) = a\int\left(x^2 - 1\right)dx + c$

$= a\left(\frac{x^3}{3} - x\right) + c$

Now $p(-3) = 0$

$\Rightarrow a\left(-\frac{27}{3} + 3\right) + c = 0$

$\Rightarrow -6a + c = 0 \quad ...(1)$

Now $\int_{-1}^{1}\left(a\left(\frac{x^3}{3} - x\right) + c\right)dx = 18$

$= 2c = 18 \Rightarrow c = 9 \quad ...(2)$

$\Rightarrow$ from (1) & (2) $\Rightarrow -6a + 9 = 0 \Rightarrow a = \frac{3}{2}$

$\Rightarrow p(x) = \frac{3}{2}\left(\frac{x^3}{3} - x\right) + 9$

sum of coefficient

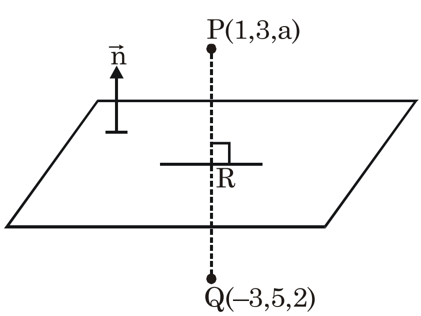
$= \frac{1}{2} - \frac{3}{2} + 9$

$= 8$

6. Let the mirror image of the point (1, 3, a) with respect to the plane $\vec{r}.\left(2\hat{i} - \hat{j} + \hat{k}\right) - b = 0$ be (–3, 5, 2). Then the value of $|a + b|$ is equal to ________.

Official Ans. by NTA (1)

Sol.



plane $= 2x - y + z = b$

$R \equiv \left(-1, 4, \frac{a+2}{2}\right) \rightarrow$ on plane

$\therefore \; -2 - 4 + \frac{a+2}{2} = b$

$\Rightarrow a + 2 = 2b + 12 \Rightarrow a = 2b + 10 \; ...(i)$

$\langle PQ\rangle = \langle 4, -2, a - 2\rangle$

$\therefore \; \frac{2}{4} = \frac{-1}{-2} = \frac{1}{a-2}$

$\Rightarrow a - 2 = 2 \Rightarrow a = 4, \; b = -3$

$\therefore \; |a + b| = 1$

7. Let $f : R \rightarrow R$ satisfy the equation $f(x + y) = f(x).f(y)$ for all $x, y \in R$ and $f(x) \neq 0$ for any $x \in R$. If Ihe function f is differentiable at $x = 0$ and $f'(0) = 3$, then

$\lim_{h\to 0}\frac{1}{h}\left(f(h) - 1\right)$ is equal to ______.

Official Ans. by NTA (3)

Sol. If $f(x + y) = f(x).f(y)$ & $f'(0) = 3$ then

$f(x) = a^x \Rightarrow f'(x) = a^x.\ell na$

$\Rightarrow f'(0) = \ell na = 3 \Rightarrow a = e^3$

$\Rightarrow f(x) = (e^3)^x = e^{3x}$

$\lim_{x\to 0}\frac{f(x) - 1}{x} = \lim_{x\to 0}\left(\frac{e^{3x} - 1}{3x} \times 3\right) = 1 \times 3 = 3$

8. Let ${}^nC_r$ denote the binomial coefficient of x^r in the expansion of $(1 + x)^n$.

If $\sum_{k=0}^{10}\left(2^2 + 3k\right){}^nC_k = \alpha.3^{10} + \beta.2^{10}$, $\alpha, \beta \in R$,

then $\alpha + \beta$ is equal to ______.

Official Ans. by NTA (19)

ETOOS Answer (Bonus)

Sol. **BONUS**

Instead of ${}^nC_k$ it must be ${}^{10}C_k$ i.e.

$\sum_{k=0}^{10}\left(2^2 + 3k\right){}^{10}C_k = \alpha.3^{10} + \beta.2^{10}$

LHS $= 4\sum_{k=0}^{10}{}^{10}C_k + 3\sum_{k=0}^{10}k.\frac{10}{k}.{}^{9}C_{k-1}$

$= 4.2^{10} + 3.10.2^9$

$= 19.2^{10} = \alpha.3^{10} + \beta.2^{10}$

$\Rightarrow \alpha = 0, \; \beta = 19 \Rightarrow \alpha + \beta = 19$

9. Let P be a plane containing the line $\frac{x-1}{3}=\frac{y+6}{4}=\frac{z+5}{2}$ and parallel to the line $\frac{x-3}{4}=\frac{y-2}{-3}=\frac{z+5}{7}$. If the point (1, –1, α) lies on the plane P, then the value of |5α| is equal to ______.

Official Ans. by NTA (38)

Sol. Equation of plane is $\begin{vmatrix} x-1 & y+6 & z+5 \\ 3 & 4 & 2 \\ 4 & -3 & 7 \end{vmatrix}=0$

Now (1, –1, α) lies on it so

$\begin{vmatrix} 0 & 5 & \alpha+5 \\ 3 & 4 & 2 \\ 4 & -3 & 7 \end{vmatrix}=0 \Rightarrow 5\alpha + 38 = 0 \Rightarrow |5\alpha| = 38$

10. Let y = y(x) be the solution of the differential equation $xdy-ydx=\sqrt{(x^2-y^2)}\,dx$, $x \ge 1$, with y(1) = 0. If the area bounded by the line x = 1, x = e^{π}, y = 0 and y = y(x) is $\alpha e^{2\pi} + \beta$, then the value of 10(α + β) is equal to ______.

Official Ans. by NTA (4)

Sol. $xdy-ydx=\sqrt{x^2-y^2}\,dx$

$\Rightarrow \frac{xdy-ydx}{x^2}=\frac{1}{x}\sqrt{1-\frac{y^2}{x^2}}\,dx$

$\Rightarrow \int \frac{d\left(\frac{y}{x}\right)}{\sqrt{1-\left(\frac{y}{x}\right)^2}}=\int\frac{dx}{x}$

$\Rightarrow \sin^{-1}\left(\frac{y}{x}\right)=\ell n|x|+c$

at x = 1, y = 0 ⇒ c = 0

y = xsin(ℓnx)

$A=\int_1^{e^{\pi}} x\sin(\ell n\,x)\,dx$

$x = e^t$, $dx = e^t dt \Rightarrow \int_0^{\pi} e^{2t}\sin(t)\,dt = A$

$\alpha e^{2\pi}+\beta=\left(\frac{e^{2t}}{5}(2\sin t-\cos t)\right)_0^{\pi}=\frac{1+e^{2\pi}}{5}$

$\alpha=\frac{1}{5}, \beta=\frac{1}{5}$ so 10(α + β) = 4